Public Economics: Practice Problem Set 2

August 5, 2017

These practice problems are meant to give you an idea of the types of questions I might ask on the final exam. To help you prepare, I have included problems that are at a slightly higher difficulty level (or that have an additional part that is slightly more difficult) than the ones you have seen on previous problem sets.

Problem 1

Consider two transfer programs that apply to individuals with relatively low earnings:

1. A transfer of \$3,000 paid to those with earnings below \$6,000 that is linearly reduced between earnings of \$6,000 and \$12,000, so that people with earnings above \$12,000 receive nothing.
2. An earned income tax credit (EITC) that is intended to encourage work. The tax credit pays nothing to those with no earnings and then linearly increases to reach a maximum benefit amount of \$4,000 for those with earnings of \$8,000. Individuals with earnings between \$8,000 and \$14,000 continue to receive the transfer of \$4,000 which is then linearly reduced between earnings of \$14,000 and \$24,000.

In addition to these two programs, incomes greater than \$15,000 are subject to a 30% income tax.

(i) What is the total effective marginal tax rate combining the effects of the two programs and the income tax at each of the following income levels: \$3,000, \$7,000, \$10,000, \$13,000, \$14,500, \$20,000, and \$25,000?

(ii) Consider an individual with utility function over consumption and leisure given by $\log(C) + 10\log(L)$. Normalize leisure to be in percentage terms, so $0 \leq L \leq 1$. Let w be the wage rate the individual earns from working full time (i.e. $L = 0$). Given the transfer programs and income tax in place, for what values of w will the individual choose to work?

(iii) Suppose that in the presence of this tax/transfer system, a consumer maximizing utility over consumption and leisure ends up with earnings of \$13,000. Would this consumer work more or less if there were no welfare programs at all?

(iv) Finally, imagine the primary earner in a family makes \$20,000 and will continue to work the same amount regardless of how much the primary earner's spouse decides to work. Does the presence of all these programs encourage or discourage the spouse to work? What does this tell you about the work incentives of the EITC?

Problem 2

Suppose an individual who lives for two periods (1,2) has a utility function over consumption this year c_1 and consumption next year c_2 given by: $u(c_1, c_2) = \log(c_1) + \frac{10}{11}\log(c_2)$. The individual

enters the first period with $21,000 and earns no further income. Assume the net interest rate is equal to 10%.

(i) What is the optimal consumption choice in each period? How much will the individual save towards consumption in the second period?

(ii) Compute the optimal consumption choice in each period when the government imposes a 5% tax on consumption in both periods.

(iii) Now imagine that instead of imposing the tax on consumption, the government taxes interest income at a rate t, with t selected so that the government collects exactly the same amount of revenue (*in present value*) as it did via the 5% consumption tax. Which of these taxes would the individual prefer – the consumption tax or the tax on interest income?

Hint: you don't need to solve for anything to answer this part of the problem.

(iv) Compute the tax rate t on interest income that yields an amount of revenue equivalent to the amount collected under the consumption tax regime.

Problem 3

Individuals live for two periods (1,2) and have utility over consumption this year c_1 and consumption next year c_2 defined by $u(c_1, c_2) = \log(c_1) + \frac{5}{6}\log(c_2)$. The individual has wealth of $12,100 as of the beginning of the first period and no further income. The net interest rate is equal to 10%. Suppose that the government imposes a 10% tax on consumption in each year, so that purchasing any given amount of consumption c costs $1.1 \cdot c$. Find consumption in both periods. Discuss how the individual's optimal savings decision differs relative to the case where there is no tax on consumption.

Problem 4

Suppose all taxpayers in the economy are identical and have preferences over consumption and leisure described by the utility function $U(C, L) = \frac{1}{2}\log(C) + \frac{1}{2}\log(L)$. Pre-tax income is given by $w \cdot (1 - L)$, where w is defined as the full-time wage rate. The government imposes a proportional tax t on earnings.

(i) Compute the marginal cost of funds (MCF). That is, compute effect on taxpayer welfare of raising an additional dollar of revenue. Show that the MCF is equal to the marginal utility of an extra dollar of income (*Hint: this is the Lagrange multiplier* λ).

(ii) Do your results in part (i) imply that there is no difference between raising revenue using a lump-sum tax (that only induces an income effect) and using a distortionary income tax?

Problem 5

Taxpayers have preferences described by $U(C, L) = \frac{1}{3}\log(C) + \frac{2}{3}\log(L)$ where L denotes the fraction of time spent on leisure. Earnings are given by $w \cdot (1 - L)$, but taxable income is $w \cdot (1 - L) - F$, where F is the amount of tax avoidance. For instance, a taxpayer can avoid tax liability by shifting earnings to non-taxable fringe benefits or by making charitable donations. While tax avoidance lowers the individual's tax burden, avoidance can only be achieved at a cost of $F^2/4b$, which is measured in consumption units. Note that b is a parameter, and $b = 0$ corresponds to a scenario where tax avoidance is infinitely costly to the taxpayer.

(i) Solve for the optimal level of tax avoidance as a function of b and t. Note that you can solve for the optimal level of tax avoidance without solving for the optimal consumption and leisure choice. How does the optimal level of tax avoidance vary with the tax rate?

(ii) Now, given the solution F^* you obtained in part (i), solve for the optimal consumption and leisure choice and compute the amount of income that the individual allows to be subject to taxation.

(iii) Does having the option to avoid paying taxes increase or reduce labor supply?

(iv) Does the presence of avoidance affect how strongly taxable income responds to changes in income tax rates? Give an economic interpretation for your response.

(v) In this problem we assumed that F only depends on the tax rate t, which allowed you to solve for the optimal level of avoidance without first solving for optimal consumption and leisure. Problems with this feature are known as a *two-stage budgeting problems*. Does this assumption make sense? What else might F depend on in a more general version of this model? What kind of implications does the simplifying assumption we've made here have for redistribution?

Problem 6

Consider a 10% tax on interest income that is imposed at the monthly frequency. Is it true that such a tax on accrual of interest income will result in the same after-tax return as a 10% on interest income collected at the annual frequency? Compute the after-tax return on a \$1 investment under each tax rule and compare your results.

Problem 7

Suppose that individuals have utility funciton $U(C, L) = C - \frac{1}{2}L^2$ where C is consumption and L is labor supply. The wage rate is w, so that pre-tax earnings are $w \cdot L$. The government imposes a proportional tax of rate t on labor income. Individuals do not have the ability to save, so consumption is equal to after-tax earnings.

(i) Solve for the individual's optimal consumption and labor supply choices as a function of the wage rate w and the tax rate t.

(ii) How does labor supply respond to a change in the tax rate levied on labor income? Decompose the impact of such a tax change into substitution and income effects and interpret the result.

(iii) Calculate the maximized level of utility (again, as a function of w and t). How does the maximized level of utility vary with the tax rate? Why is this the case?

(iv) Write down the government's revenue from the tax on labor income as a function of t and w.

(v) Suppose the government uses all of the revenue R collected from the tax on labor income to fund a transfer to the individual. Note that this means that actual revenue is $R = t \cdot wL$, even though the individual acts as if they are receiving a lump-sum transfer. That is, the individual does not take into account the fact that their taxes are paying for the transfer they ultimately receive. Given this environment, at what value of t will utility be maximized? How does utility change as t increases? Why is this the case even though transfers and taxes exactly offset each other?

Problem 8

Does capital income taxation always encourage entrepreneurs to invest in risky projects? If yes, provide examples to illustrate your response. If not, provide a counterexample.

Problem 9

Suppose there are two kinds of taxpayers: those who are self-employed and those who are employees at a company. Income for both types is given by y, and each type of taxpayer faces the same proportional tax rate $t = 0.3$. Both types of workers decide how much of their tax liability to evade. Denote f the dollar amount of the fine (if caught) for tax evasion per dollar of tax evaded. Given a taxpayer's chosen amount of evasion E, the dollar amount of taxes paid if evasion is successful is $t \cdot (y - E)$. If a taxpayer is caught evading taxes, the amount of tax paid is $t \cdot y$ and the taxpayer has to pay a penalty of $f \cdot E$.

(i) Imagine the self-employed and employees are equally likely to be caught evading taxes. How high should the government set the probability of detecting evasion in order to completely discourage evasion?

(ii) Now suppose that during an audit it is easier for the tax authority to detect evasion by employees than by self-employed taxpayers. In particular, audits are sucessful at discovering cheating 80% of the time for employees, but successful only in 50% of cases for the self-employed. How often should the government audit each type of taxpayer if its goal is to completely discourage evasion?

(iii) The audit probabilities you computed in part (ii) are very high relative to estimates for audit probabilities in the U.S. ($< 1\%$), even though the other parameters we have set in this model are empirically plausible. This suggests that in practice, given such a low audit probability we should observe a lot of tax evasion. Explain what is missing from this model that could rationalize the relatively small tax gap of 15-17% reported by the IRS in recent years.

(iv) Return to the setup in part (i) where both types of taxpayers are equally likely to be caught evading. Augment the model so that the probability of detecting evasion is no longer constant and now varies with the level of tax evasion. Assume the probability of discovering evasion is $p(E) = E/100000$, and when $E > 100000$, tax evasion is discovered with certainty (we ignore the possibility of negative evasion or overreporting in this model). Compute a taxpayer's optimal amount of evasion as a function of the penalty rate f.

(v) How does the level of tax evasion you computed in part (iv) depend on income? How would you modify the model, as it exists in part (iv), to make it more realistic?

Problem 10

Taxpayers have fixed pre-tax income y. Given the chosen amount of tax evasion E, the tax paid if evasion is successful is given by $t \cdot (y - E)$. If a taxpayer is caught cheating on their taxes, they pay a fine $f \cdot tE$ on top of their tax liability $t \cdot y$. In this problem, we assume tax evasion carries an additional cost to the individual given by $E^2/2y$. We interepret this cost as being due to a combination of psychological discomfort of doing something wrong, stress caused by the idea of being caught, exposure to risk, monetary expenses, etc. Assume that the taxpayers are audited with probability p and that an audit automatically uncovers tax evasion.

(i) For each of the following three values of p: 0.01, 0.02, 0.1, how high should the government set the penalty rate f to stop taxpayers from evading altogether?

(ii) What is the optimal level of tax evasion as a function of the parameters of the problem (i.e. without assuming any particular value for f or p)?

(iii) Define the elasticity of taxable income as

$$\varepsilon = \frac{d(y - E)}{dt} \cdot \frac{1 - t}{y - E}$$

Compute this elasticity and comment on how it varies with the policy parameters p and f.

Problem 11

A family of four with income \$100,000 has a lower ability to pay than a single individual with the same income of \$100,000. Should the family of four and the single individual face different tax liabilities according to the Haig-Simons criterion? How does the current U.S. tax system address this disparity in ability to pay along the household size dimension?

Problem 12

Explain why a government might prefer to provide welfare benefits in-kind rather than a cash benefit to *limit program participation*. Could a government offer a welfare program with cash benefits and use other methods to limit program participation? Give a real-life example specific to the U.S. to illustrate your response.

Problem 13

Suppose there is one taxpayer with a talent level that exceeds the talent level of all other taxpayers in the economy. Using the optimal top marginal tax rate formula discussed in lecture, explain why the most-talented taxpayer should face a 0% tax rate. If talent can be arbitrarily high (i.e. there is not just one most-talented individual), what other considerations does a government need to take into account when setting the top marginal tax rate?

Problem 14

Besides the differences in tax instruments available to the government across the two models, how do the government's motivations differ across the optimal commodity tax and optimal income tax models? In particular, why is there no role for redistribution in the optimal commodity tax framework we studied in lecture? Given that there is no role for redistribution in the optimal commodity tax problem, what is the tradeoff the government faces when deciding how to allocate tax collection across two taxable goods?

Problem 15

The figure below presents the benefits schedule for the EITC in 2017. Explain what are the theoretical predictions for the effect of the EITC on labor supply. In particular, verify each of the following statements as TRUE, FALSE, or UNCERTAIN, and give an explanation for each of your answers. Recall the terminology we used to describe labor supply: intensive margin refers to how many hours the individual decides to work, whereas extensive margin refers to whether the individual chooses to work at all.

(i) The EITC increases labor supply at the extensive margin.

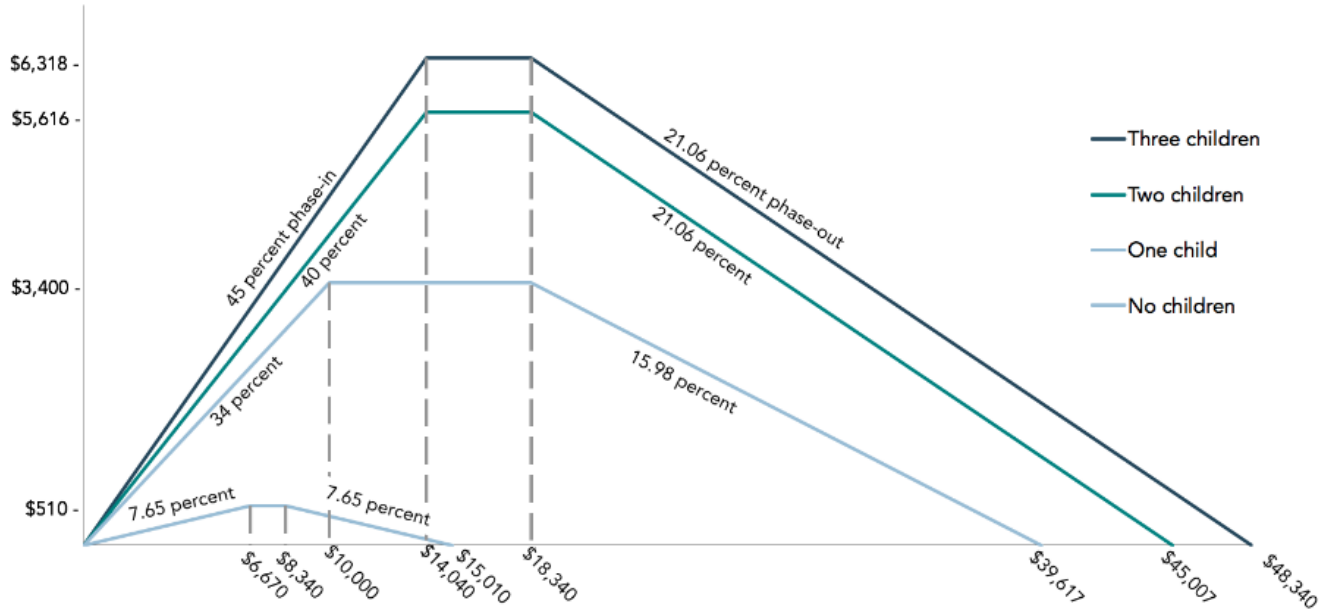
(ii) The EITC increases labor supply at the intensive margin for a single parent with one child who would have earned $5,000 in the absence of the EITC (the phase-in region).

(iii) The EITC increases labor supply at the intensive margin for a single parent with one child who would have earned $12,000 in the absence of the EITC (the plateau region).

(iv) The EITC increases labor supply at the intensive margin for a single parent with one child who would have earned $25,000 in the absence of the EITC (the phase-out region).

FIGURE 1
Earned Income Tax Credit
2017

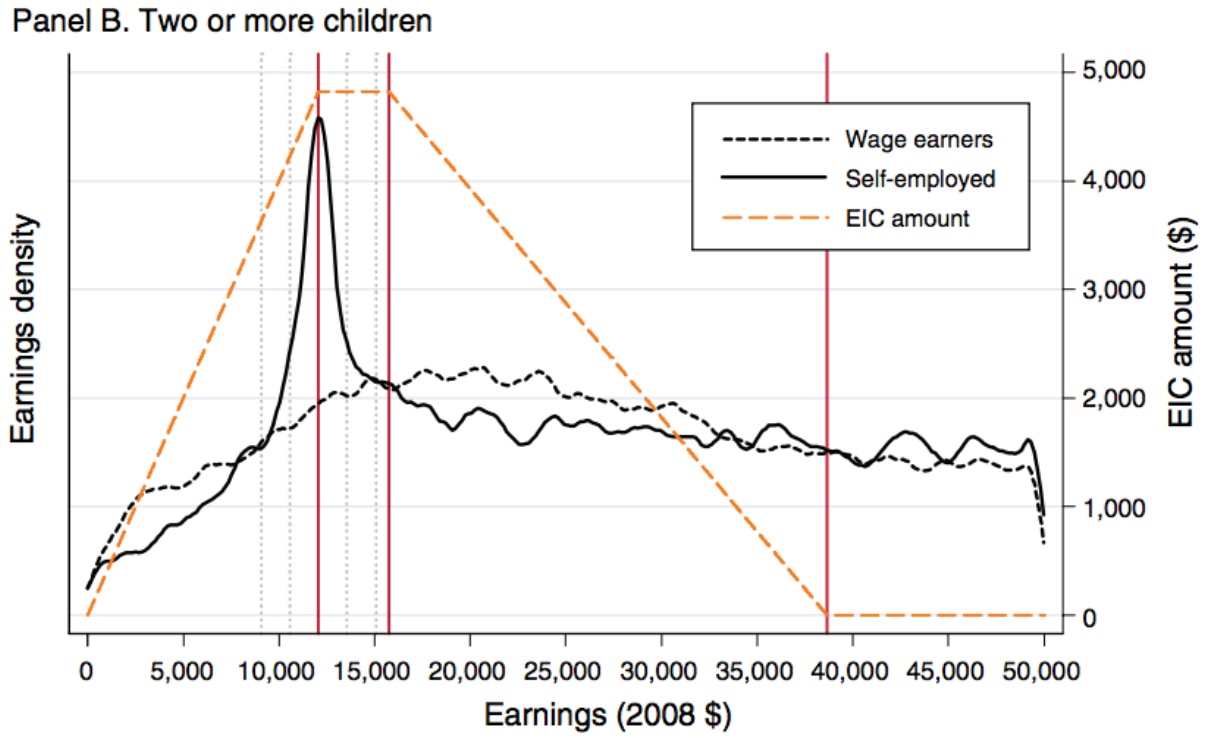
Credit amount



Source: Tax Policy Center, IRS Rev. Proc. 2016-55.
Note: Assumes all income comes from earnings. Amounts are for taxpayers filing a single or head-of-household tax return. For married couples filing a joint tax return, the credit begins to phase out at income $5,590 higher than shown.

Problem 16

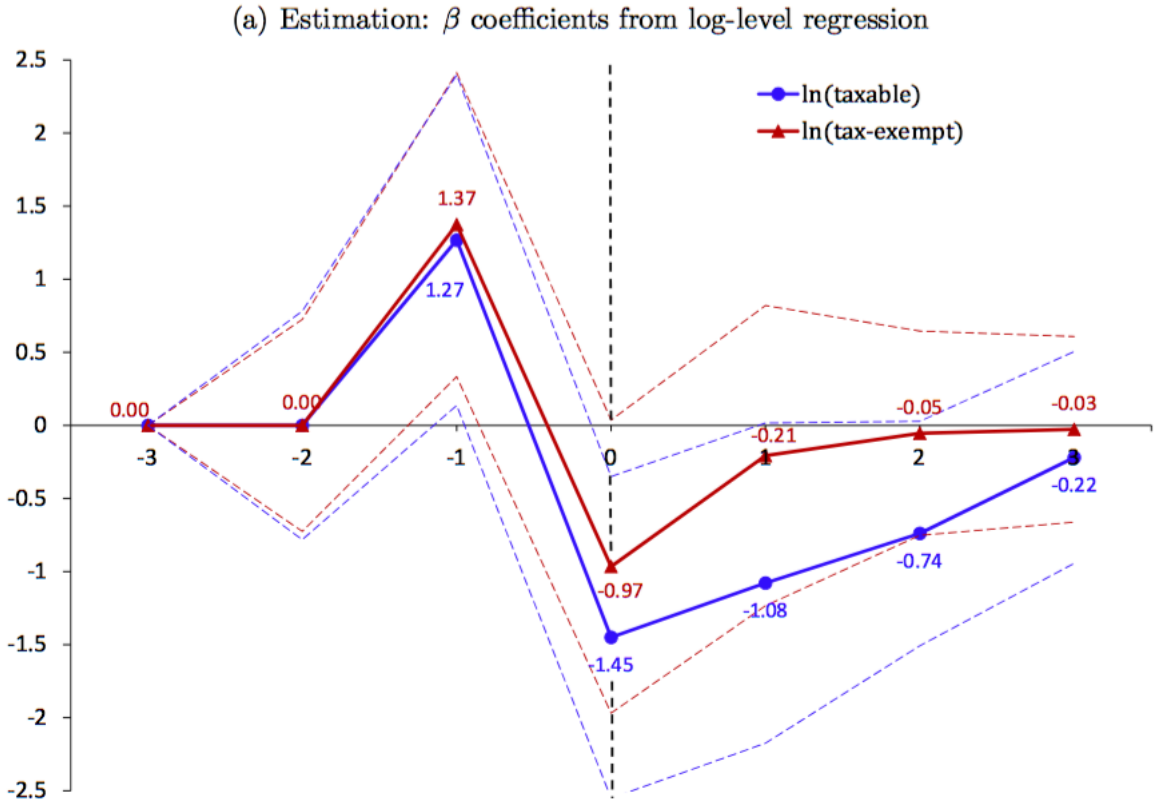
Explain what the figure below tells us about the elasticity of taxable earnings for the self-employed compared to wage earners (i.e. employees). Based on your answer, would the efficiency cost of an income tax (set at any tax rate) be higher or lower for self-employed taxpayers? Why is there this difference in the labor supply response of the self-employed?



Source: Saez (2010), "Do Taxpayers Bunch at Kink Points?" *American Ecnomic Journal: Economic Policy*

Problem 17

Consider the figure below from Baker et al. (2017), "Shopping for Lower Sales Taxes," which shows how expenditures (in percentages) on goods change in the months before and after a 1% sales tax increase (at time 0). Explain how the figure illustrates a particular strategy consumers use to avoid some of the burden of anticipated future sales tax increases. Why do we observe in the figure that expenditures on both taxable and tax-exempt goods increase in the month prior to a tax change?



Baker, Johnson, & Kueng (2017), "Shopping for Lower Sales Tax Rates," *mimeo*. Northwestern University