COMING IN AT A TRICKLE: THE OPTIMAL FREQUENCY OF PUBLIC BENEFIT PAYMENTS

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- Governments offer public transfer payments to qualifying citizens which pay out at regular intervals (UBI, UI, Social Security/SSI, SNAP/WIC)
- Example: pay cycle lengths for 36 OECD countries w/universal public pensions
 - ▶ 31 operate on (semi-)monthly systems
 - ▶ 2 every two weeks (Australia/New Zealand)
 - 2 annually (Iceland/Ireland)
 - ▶ 1 every two months (Japan)
- Also calendar variation within countries ("5th Friday" or "birthday" rules)
 - ▶ Pro: fixing a calendar date for payments good if otherwise non-salient
 - ▶ Con: can magnify welfare losses from self-control problems, liquidity constraints, etc.
- Extant research examines design of these programs in terms of limiting moral hazard, financing, redistributive consequences

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ONE BIG (NEST) EGG, OR MANY SMALL ONES?



What we do

Policy question

How should governments set the frequency of benefit payments?

- Introduce sufficient statistics approach to determining optimal pay frequency
 - ▶ Regulator faces tradeoff: ↑ frequency ⇒ ↑ admin costs and ↓ welfare loss from consumption non-smoothing
 - Model can flexibly accommodate various behavioral frictions
 - ► Complements work on pay timing from employer's POV (Parsons & Van Wesep 2013)
- Empirical application to national Japanese Pension System (JPS)
 - ► High-frequency retail scanner data linked to loyalty point cards
 - ► Can separate prices from quantities to isolate consumption, retailer responses
 - ► Lower-frequency payments feasible alternative to raising normal retirement age

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Basic modeling framework

 \bullet Govt. picks T to minimize welfare loss subject to balanced budget

Full model

$$\min_{T} \left\{ -p \cdot \lambda(T) + \left(p \cdot b(T) + \mu(T) \right) \right\}$$

ullet Govt. sets length of pay cycle T^* to equate the marginal reduction in the welfare loss to marginal cost of reducing T

$$\underbrace{p \cdot \lambda'(T^*)}_{\text{marginal benefit}} = \underbrace{\mu'(T^*) + p \cdot \overline{B}}_{\text{marginal cost}}$$

- Depends on fraction of recipients p, the average daily benefit amount \overline{B} , slope of welfare loss $\lambda'(T)$ and cost function $\mu'(T)$
- Key challenge: admin costs and welfare losses not directly observed
 - lacktriangle Exploit local exposure to 1980s pension system reform which moved T=90
 ightarrow 60
 - ► Admin costs increase by 4% ⇒ fairly flat cost function



What underlying behaviors could generate non-smoothing?

- Liquidity constraints: Zeldes (1989); Broda & Parker (2014); Baker (2018) By age By quality
 - ▶ No retirement consumption drop + similar responses by income based on store quality
- Near-rationality (Kueng 2018): welfare loss is small relative to permanent income
- Onsumption commitments: timing of bills matches timing of income
 - ► Chetty & Szeidl (2007); Vellekoop (2018) focus on mortgage payments
 - ► We use data on predominantly perishable, non-durable goods spending

 Full literature Retirement puzzle Commitment
- Present-bias: approx. log-linear decline in consumption in between paydays
 - ▶ Shapiro (2005); Huffman & Barenstein (2005); Mastrobuoni & Weinberg (2009)
- Mental accounting: people behave as if they have a license to spend on payday
 - ▶ Thaler (1999); Gelman et al. (2014); Olafsson & Pagel (2018); Farhi & Gabaix (2020)

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INTUITION: OPTIMAL FREQUENCY UNDER TWO "NAIVE" INTERNALITIES

- Quasi-hyperbolic discounting: with $\beta \cdot \delta$ discounting, as $\beta \to 1$, consumption declines almost linearly over pay cycle at rate $f(t) = \nu \cdot t$ Details Figure
- @ Mental accounting ("payday liquidity"): extra spike in consumption on payday x(T)
 - For log utility, inc. frequency of payments $(T\downarrow)$ can improve welfare if...

- ullet $\lambda'(T)$ quantitatively very similar regardless of underlying behavioral mechanism
 - ► True for a wide range of parameter values (i.e. not just in our setting)
- Calibrated model offers support for monthly payment schedules

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EMPIRICAL SETTING: JAPANESE PUBLIC PENSION SYSTEM

- Largest public pension fund in the world (\$474 bil. paid out annually)
- Structure similar to U.S. Social Security
- Background Sample profile
- ▶ Early retirement age 60, normal retirement age 65, late retirement until age 70
- Paydays scheduled for the 15th of each even month
 - ► If scheduled date falls on Saturday, Sunday, or public holiday, moved to first previous non-holiday weekday ⇒ random variation in pay cycle length
- Three other advantages to the Japanese setting:
 - **1** Difficult to buy in bulk due to storage costs \implies spending \approx consumption
 - Universal health insurance \iff little need to save for uncertain medical expenses
 - **1** Pension payments account for > 80% of income for recipients (survey evidence)

TIMESTAMPED RETAIL DATA ON SHOPPER SPENDING HISTORIES

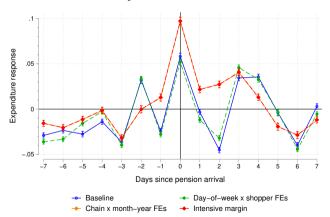
- Hourly retail scanner data from Japanese marketing research firm
 - Covers regional grocery store chains for 2011-14
 - Prices and quantities at barcode level
- Classification
- Shoppers' purchase history connected to loyalty point card
 - ▶ Basic demographic info: store/chain ID, Census region, gender, and age (MM/YYYY)
 - Use age to determine pension eligibility (intent to treat)
 - ► Scale up by claiming probability from retirement surveys (95% claim by age 65)
- Apply restrictions to obtain set of regular (weekly) shoppers and stores visited
 - ► Final sample: 511 stores spread across 21 chains, 416,726 unique shopper IDs
 - ▶ 38% above normal retirement age, 51% reach early retirement age





Baseline: High-frequency event study around paydays

$$\frac{X_{i,c,t}}{\overline{X}_{i,c}} = \sum_{j=-7}^{+7} \beta_j \cdot Payment_{i,t+j} + \delta_{dow} + \phi_{wom} + \psi_{my} + \xi_h + \eta_i + \epsilon_{i,c,t}$$
 (2)



- Hetero. treatment effects due to preferences over store brands
- Use 10% \uparrow to calibrate behavioral frictions underlying $\lambda(T)$
 - Same as spike in spending on perishables Robustness
 - $\implies \nu = 0.2\% \text{ daily consumption} \\ \text{decline in between paydays}$
- Spending concentrated in splurge goods like prepared foods (22%↑) and alcohol (28%↑)

Calendar variation in intervals to test for pent-up demand

$$\frac{X_{i,c,t}}{\overline{X}_{i,c}} = \beta \cdot Payday_t \times Length_{t \in p} + \delta_{dow} + \phi_{wom} + \psi_{my} + \xi_h + \eta_i + \epsilon_{i,c,t}$$
(3)

	(1)	(2)	(3)	(4)	(5)	(6)
$Payday \times Length$	0.0010***	0.0013***	0.0001^{+}	-0.0347***	-1.0550***	-0.1910***
	(0.0000)	(0.0000)	(0.0006)	(0.0010)	(0.0297)	(0.0448)
$Payday \times Length^2$			0.0000	0.0006***	0.0353***	0.0058***
			(0.0000)	(0.0000)	(0.0010)	(0.0015)
$Payday \times Length^3$					-0.0003***	-0.0000***
					(0.0000)	(0.0000)
Time FEs	~	~	~	~	~	~
Intensive margin		✓		✓		~
$\widehat{x}(T=60)$	0.060	0.078	0.058	0.078	0.276	0.096
Joint F-test (p-value)	-	-	0.000	0.000	0.000	0.000
N	210,469,638	86,632,913	210,469,638	86,632,913	210,469,638	86,632,913
$\operatorname{Adj.} olimits R^2$	0.025	0.329	0.025	0.329	0.025	0.329

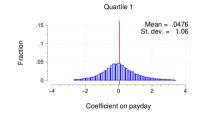
 $ightarrow \Rightarrow 0.13$ p.p. inc. in payday spending for each extra day in pay cycle p

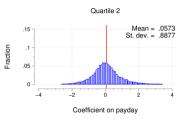


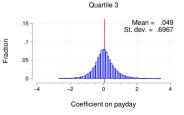


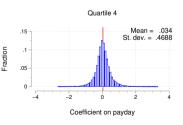
LIMITED EVIDENCE IN FAVOR OF LIQUIDITY/NEAR-RATIONALITY STORY

$$X_{c,t}^{i}/\overline{X}_{i,c} = \beta^{i} \cdot Payday_{t} + \delta_{dow} + \phi_{wom} + \psi_{my} + \xi_{h} + \epsilon_{t}^{i}$$









- Run separate time series regression for each shopper age ≥ 65
- Sort $\widehat{\beta}_i$ by i's quantile of avg. total pay cycle expenditures
- Total spending reasonable proxy for permanent income (Kueng 2015,18)
- Stable avg. payday response across PI dist.





SEPARATING RETAILER RESPONSES FROM "SPLURGE" SPENDING

- Formally decompose observed store-level daily inflation $\Delta \Phi_{s,t}$ into...
- **Consumer variety effects:** buying more barcodes in set of commonly purchased goods Ω^*
- ightharpoonup Consumer substitution effects: quality upgrading Ω^{new} vs. Ω^{old} Evidence
- ▶ Retailer response: sales or change in discount rate or regular price within Ω^*
- Punchline: quantitatively small retail pricing response, driven by targeted temporary sales
 - For above (below-) median priced goods, payday sales 1.5 p.p. less (more) likely, with 1 p.p.
 - Effect on $\Delta\Phi_{s,t}$ quantitatively important only for prepared foods
 - ▶ Uniform pricing across stores (DellaVigna & Gentzkow 2019) → use chain × time FEs
 - Robust to choice of temporary sales filter (Kehoe-Midrigan or Nakamura-Steinsson)



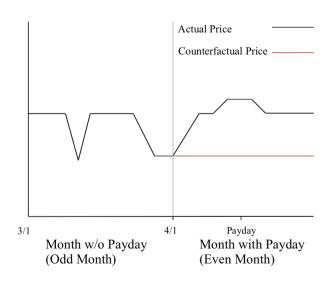
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- ullet Formally decompose observed store-level daily inflation $\Delta\Phi_{s,t}$ into...
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- Retailer response: sales or change in discount rate or regular price within Ω^*
- **Punchline:** quantitatively <u>small</u> retail pricing response, driven by targeted temporary sales strategy on payday Filters
 - For above (below-) median priced goods, payday sales 1.5 p.p. less (more) likely, with 1 p.p. less (more) generous discounts

 Distributions
 - Effect on $\Delta\Phi_{s,t}$ quantitatively important only for prepared foods
 - lacktriangle Uniform pricing across stores (DellaVigna & Gentzkow 2019) \longrightarrow use chain imes time FEs
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ILLUSTRATION: COUNTERFACTUAL "LAST PRICE" INDEX

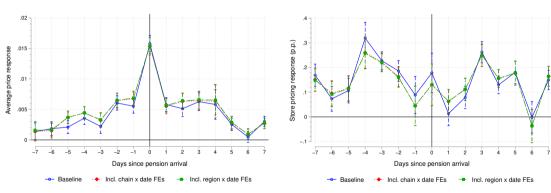


- **Idea:** hold fixed barcode-level p to isolate demand changes
- Two-step procedure:
 - Event study w/outcome $\Delta\Phi_{s,t}$
 - 2 Same event study w/outcome $\Delta\Phi_{s,t}^{last}$ and take difference in coefficients
- Check robustness to measures of "last" prices Check
 - Last month index (diagram)
 - Last week index using prices in week before payday

NO CLEAR RETAILER PRICING RESPONSE AROUND PAYDAY

Store level average price index $\Phi_{s,t}$

Retailer response = $\Delta\Phi_{s,t} - \Delta\Phi_{s,t}^{last}$



- $\bullet < 10\%$ of store-level inflation around payday due to retail price changes
- \implies "menu costs" sufficiently large that $P(T) \equiv P$ is a reasonable simplification

NATURAL EXPERIMENT TO ESTIMATE SLOPE OF ADMIN COST FUNCTION

$$\underbrace{p \cdot \lambda'(T^*)}_{\text{marginal benefit}} = \underbrace{\mu'(T^*) + p \cdot \overline{B}}_{\text{marginal cost}}$$

- Use 1988 reform to the JPS which **only altered pay frequency** without changing eligibility criteria or generosity of benefits
 - ▶ Transitioned from payments every 3 months to every 2 months $(T = 90 \rightarrow 60)$
- Identification: exploit fact that municipal budgets differentially exposed to admin costs of the reform depending on whether they have one of 312 local JPS branch offices
 - ▶ Local offices run day-to-day operations of JPS w/o managing pension funds
 - Admin costs: applications, reconciling benefits, confirming eligibility, investigating fraud
- Data: local public spending on elderly welfare benefits from PM's Cabinet Office
 - ▶ National welfare programs, so per capita non-JPS spending differenced out

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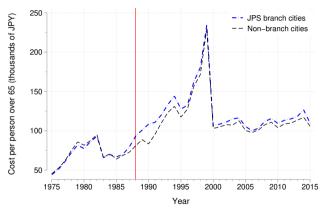
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DID analysis $\implies \approx 4\% \uparrow$ in admin costs from moving $T = 90 \rightarrow 60$

$$\log \mu_{j,t} = \beta \cdot Branch_j \times Post_t + \gamma_j + \delta_t + \epsilon_{j,t}$$
(4)

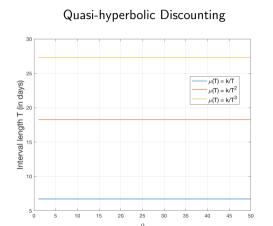


Notes: Municipal spending in thousands of real 2012 JPY on administering the pension system and elderly welfare benefits divided by the number of persons over age 65 residing in the municipality.

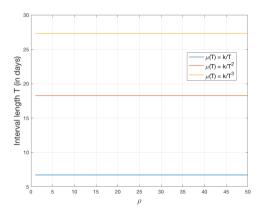
- Small uptick in costs among JPS branch cities starting in FY 1987
- Branch office cities more populated, but similar per capita spending on elderly
- At most, covariate-adjusted uptick in costs of 7%

Map + balance Table

Optimal frequency flat w.r.t. inverse IES ho

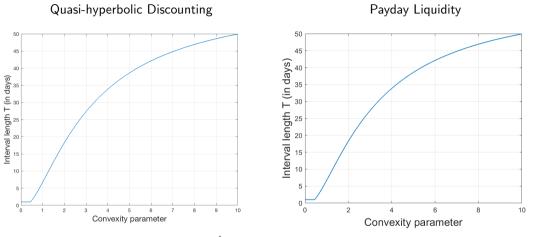


Payday Liquidity



- T^* very weakly decreasing in ρ (out to 6 decimals), regardless of type of internality
- Logic: welfare loss $\lambda(T^*)$ varies a lot with ρ , but marginal loss $\lambda'(T^*)$ does not

OPTIMAL FREQUENCY CONCAVE W.R.T. CONVEXITY OF ADMIN COSTS



- Calibration: suppose $\mu(T)=\kappa_\ell/T^\ell$ and for each power ℓ set κ_ℓ so that $\mu(60)$ equals the administrative service costs reported for FY 2011
- ullet For $\ell < 0.5$ daily frequency ("continuous trickle") would be optimal



IMPLICATIONS FOR PENSION ELIGIBILITY AGE REFORMS

More than 1.2 million march in France over plan to raise pension age to 64

Protesters aim to 'bring France to standstill' as President Macron struggles to delay retirements by 2 years



- Many countries with pay-as-you-go systems raising retirement age to cut costs as birth rate declines
 - Japan: phased increase in NRA from 65 to 70
 - ► France: April 2023 increase from 62 to 64 → protests!
 - ▶ UK: phased increase from 65 to 67 between 2020 to 2028
- Alternative: cut admin costs by sending same pension amount but divided into fewer payments

Source: The Guardian (March 7, 2023).

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Cost comparisons: eligibility age vs. payment frequency reforms

- Consider April 2021 Japanese plan to raise NRA for flat-rate pensions from 65 to 70
- ullet Counterfactual: what would be the increase in pay cycle length $\Delta T>0$ required to reduce costs by an equivalent amount to raising the NRA?
- Answer: equivalent to moving along the cost function μ from T=30 (our upper bound optimum) to T=45, or payments every 6.5 weeks
 - ▶ 36.12 billion JPY in initial annual savings at stake Details
 - ightharpoonup Assume distribution of claiming ages from Japanese Pension Survey in 2021 \longrightarrow valid if moral hazard is minimal
 - ▶ Caveat: ignores the revenue side, so opportunity cost of keeping NRA fixed might be greater
- In practice, gains to increasing T and NRA simultaneously ("double dividend")



▶ Intuition: govt. FOC easier to satisfy when fraction eligible declines $\implies T^* \uparrow$

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- Result
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Conclusion: Support for prevalence of monthly pay cycles

- First paper to consider payment frequency as a policy parameter
- Framework is simple, but can be applied to any country and public benefit program with data on costs and high-frequency recipient behavior
- In the empirical application, we show:
 - Large spike in expenditures on payday which appears to be unrelated to liquidity proxies
 - ★ Mental accounting, or consumer type switching within pay cycle
 - ▶ Limited evidence of retailer price discrimination ⇒ menu costs are large
 - ► Calibrated model yields optimal frequency ≤ 1 month
- Variety/substitution effects are important drivers of prices during peak demand periods

 — implications for debate on inflation due to COVID-19 stimulus payments
- Lowering pension frequency might be more attractive than raising retirement age

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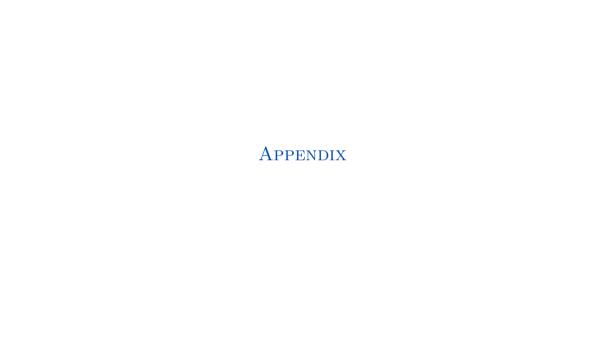
 → implications for debate on inflation due to COVID-19 stimulus payments
- Lowering pension frequency might be more attractive than raising retirement age

Conclusion: Support for prevalence of monthly pay cycles

- First paper to consider payment frequency as a policy parameter
- Framework is simple, but can be applied to any country and public benefit program with data on costs and high-frequency recipient behavior
- In the empirical application, we show:
 - Large spike in expenditures on payday which appears to be unrelated to liquidity proxies
 - ★ Mental accounting, or consumer type switching within pay cycle
 - ► Limited evidence of retailer price discrimination ⇒ menu costs are large
 - ► Calibrated model yields optimal frequency ≤ 1 month
- Lowering pension frequency might be more attractive than raising retirement age

THANK YOU!





Contributions to the literature



• Consumer responses to the timing of (regular) payments:

▶ Stephens (2003,06); Shapiro (2005); Dobkin & Puller (2007); Mastrobuoni & Weinberg (2009); Foley (2011); Stephens & Unayama (2011); Evans & Moore (2012); Gelman et al. (2014); Olafsson & Pagel (2018); Vellekoop (2018); Baker (2018); Baugh & Wang (2021); Baugh & Correia (2022); Zhang (2022); Gross, Layton, Prinz (2022)

• Motivations for consumption non-smoothing behavior:

Zeldes (1989); Thaler (1999); Huffman & Barenstein (2005); Chetty & Szeidl (2007); Broda & Parker (2014); Farhi & Gabaix (2020); Kueng (2015,18); Parker (2017); Hastings & Shapiro (2018); Chevalier & Kashyap (2019); Baugh, Ben-David, & Parker 2021

• Retailer pricing during peak demand periods:

 Warner & Barsky (1995); MacDonald (2000); Chevalier, Kashyap, Rossi (2003); Nevo & Hatzitaskos (2006); Hastings & Washington (2010); Goldin, Homonoff, & Meckel (2022)

• Retirement consumption puzzle:

▶ Bernheim, Skinner, & Weinberg (2001); Aguiar & Hurst (2005); Battistin et al. (2009); Stephens & Unayama (2012); Agarwal, Pan, Qian (2015); Olafsson & Pagel (2020)

Public Pension Benefit System in Japan

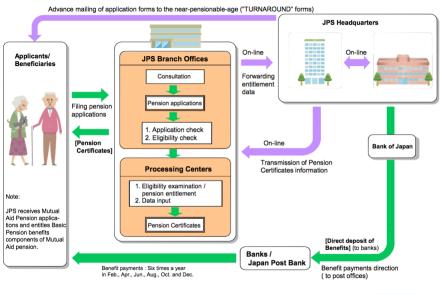


- Japan's mandatory public pension system (JPS) has two tiers
 - ▶ National pension (NP): flat-rate pension w/compulsory coverage for residents age 20-59
 - ▶ Employee pension insurance (EPI): earnings-related pension with compulsory coverage for those employed full-time by private company with ≥ 5 workers
 - ▶ NP and EPI implemented jointly as one system (i.e. same payment timing)
- Other features related to payment amounts
 - Earnings test: if working beyond age 65, EPI benefit reduced or suspended if monthly EPI payment + wages > 460,000 JPY
 - ▶ Normal retirement at age 65, with early (60-64) or deferred (66-70) collection possible
 - Not very generous compared to other OECD countries: 2012 full NP amount was 780,100 JPY (\approx \$7,800) per year for 40 years of contributions

How are JPS benefits paid out?

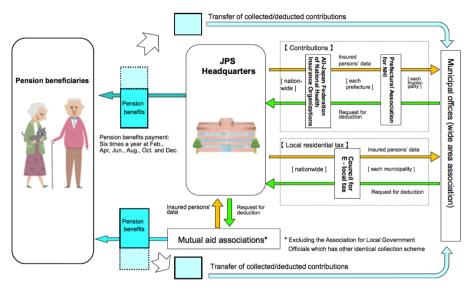


- Both NP and EPI payments are distributed regularly on the 15th of even months
- If scheduled benefit delivery date falls on a Saturday, Sunday, or public holiday, it is moved to the first previous non-holiday weekday
- 22 delivery dates in our sample time period: 15 on the 15th, 4 dates moved to the 14th, and 3 moved to the 13th
- Payments usually arranged via bank transfer when pensioners submit a form to local city office to begin claiming benefits
 - ▶ Local city offices not directly involved in remitting payments
 - ▶ But involved in processing applications and withholding taxes from pension payments



Source: "Japan Pension Service and its Operation", Japan Pension Service, April 2017 Report.

Main deck

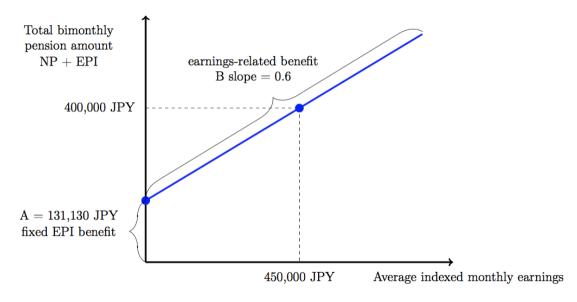


Source: "Japan Pension Service and its Operation", Japan Pension Service, April 2017 Report.



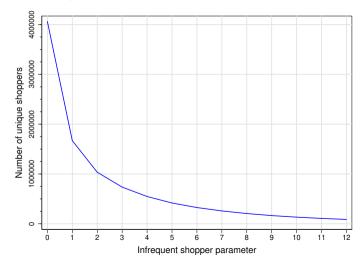
SAMPLE BENEFIT SCHEDULE







Regular shoppers with $\geq k$ store visits each month

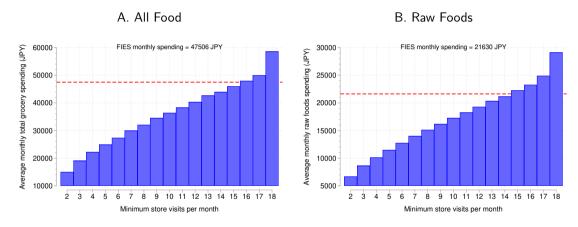


- Baseline: always restrict to weekly shoppers $(k \ge 4)$
- Defining regular shopper panel helps pin down consumption
- Non-regular shoppers either have access to storage or do their spending elsewhere
- Check results similar for k=2 (every other week)

Retail Expenditures Summary Statistics

	All Goods	Raw Foods (Perishables)
Avg. daily expenditures (JPY)	2,603	1,149
Avg. monthly expenditures (JPY)	35,184	14,048
Avg. number of monthly trips	13.0	11.5
Avg. periodicity	2.0	2.3
% Female shopper	65.7%	65.5%
% Early retirement age	45.5%	45.5%
% Normal retirement age	31.2%	31.5%
# Stores	511	510
# Shoppers	409,439	416,726

Main deck



- Match FIES monthly spending for shoppers who go to store every other day, on average
- Daily spending non-monotonic in trip frequency use weekly shoppers as baseline

One-digit goods category classification system



One-digit Category	Two-digit Category	Four-digit Categories			
Fresh fruits & vegetables	Fresh fruits	seasonal fruits, imported fruits,			
		assorted fruits, fruit-related products,			
	Fresh vegetables	leafy veg., stalk veg., root crops,			
		edible plants, edible seeds, mushrooms,			
		germinated veg., assorted veg.			
Processed fruits & vegetables	Processed fruits	frozen fruits, cut fruits,			
	Processed vegetables	boiled veg., frozen veg., cut veg.			
Fresh fish	Fresh fish	round items, filet,			
		shellfish, assorted fish			
	Sashimi	brick form, sashimi, tataki,			
		raw fish, assorted fresh fish			
Preserved fish products	Salted & dried fish	boiled fish, frozen fish,			
		seasoned fish, pickled fish,			
		salted fish, dried fish,			
		fish eggs, seaweed			
Raw meat & poultry	Beef	wagyu, domestic beef, imported beef,			
	Pork	domestic pork, imported pork			
	Chicken	domestic chicken, imported chicken,			
		brand name chicken, duck meat			
	Meat varieties	lamb, horse meat, minced meat,			
		offal, raw meat, eggs, dairy products			
Grains	Cereals	powder, rice, mochi,			
		raw noodles, dough, bread, cereal			

Two-digit Category	Four-digit Categories				
Seasonings	cooking oil, spices, condiments,				
	spread/dips, toppings, rice seasoning				
Dry produce	dried fish, dried fruits				
Processed food	pickled items, processed fish, pastes,				
	cooked beans, processed meats				
Instant foods	cup noodle, instant soup,				
	frozen foods, sealed rice pouch				
Semi-prepared dishes	fried, simmered, grilled,				
	Japanese, Western, Chinese				
Side dishes	fried, grilled, grilled eel,				
	Japanese, Western, Chinese				
Bento	cooked rice, sushi, bread dishes, noodle dishes				
Confectionery	toppings, jelly/pudding, ice cream,				
	frozen confections, candies/cookies, rice crackers				
Beverages	coffee/tea, milk-based drinks,				
	vegetable/fruit drinks, soft drinks				
Alcohol	beer, liqueurs, wine				
	liquor, sake				
Tobacco	tobacco				
Other	flowers, gifts/confections,				
	kiosk goods, service counter goods				
	Seasonings Dry produce Processed food Instant foods Semi-prepared dishes Side dishes Bento Confectionery Beverages Alcohol Tobacco				

Payday Spending Responses by Margin and Goods Category



Category	Overall	Incl. Chain FEs	Intensive	Extensive	
All goods	0.059***	0.099***	0.096***	0.001***	
	(0.001)	(0.002)	(0.002)	(0.000)	
Raw foods	0.053***	0.093***	0.093***	0.001	
	(0.001)	(0.002)	(0.002)	(0.000)	
Prepared foods	0.079***	0.212***	0.219***	0.002***	
	(0.003)	(0.007)	(0.006)	(0.000)	
Sweets/desserts	0.069***	0.166***	0.167^{***}	0.006***	
	(0.003)	(0.007)	(0.007)	(0.000)	
Alcohol	0.137***	0.275***	0.281***	0.004***	
	(800.0)	(0.053)	(0.051)	(0.000)	
Fresh produce	0.044***	0.077***	0.076***	0.001*	
	(0.002)	(0.003)	(0.003)	(0.000)	
Fresh fish	0.060***	0.226***	0.225***	0.001**	
	(0.003)	(0.009)	(0.009)	(0.000)	
Meat & poultry	0.049***	0.141***	0.132***	0.002***	
	(0.002)	(0.005)	(0.004)	(0.000)	

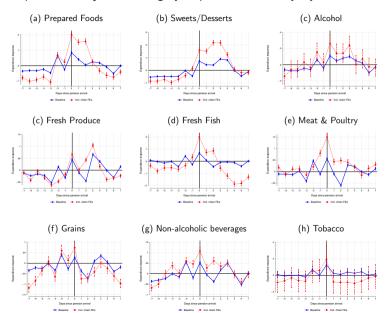
Category	Overall	Incl. Chain FEs	Intensive	Extensive	
Grains	0.024***	0.092***	0.073***	0.001^{+}	
	(0.003)	(0.009)	(800.0)	(0.000)	
Non-alcoholic beverages	0.048***	0.110***	0.101***	0.005***	
	(0.002)	(0.006)	(0.006)	(0.000)	
Tobacco	0.137***	0.135	0.140^{+}	0.001	
	(0.026)	(0.086)	(0.079)	(0.001)	
Processed fruits/vegetables	0.051***	0.180***	0.120**	0.002***	
	(0.006)	(0.039)	(0.037)	(0.000)	
Preserved fish	0.028***	0.064***	0.060***	0.002***	
	(0.003)	(0.011)	(0.010)	(0.000)	
Other processed foods	0.056***	0.107***	0.102***	0.003***	
	(0.002)	(0.003)	(0.003)	(0.000)	

Spending concentrated in discretionary goods categories

Notes: Each cell in the table is the coefficient on Payment from a separate regression within a particular expenditure subcategory. Overall refers to the spending response and including shopper-day observations of zero expenditures. The second column indicates how our point estimates of the overall spending response changes when we include store chain fixed effects. The dependent variable in the intensive margin regressions is expenditures on a store visit relative to average daily expenditures. The dependent variable in the extensive margin regressions is a dummy for whether the shopper makes a purchase on a given date. In each regression, we winsorize the top 1% of total daily expenditures. Robust standard errors clustered by shopper ID in parentheses. ****p < 0.001, **p < 0.01, **p < 0.05, +*p < 0.05, +*p < 0.01

Response of Major Subcategory Expenditures to Payday



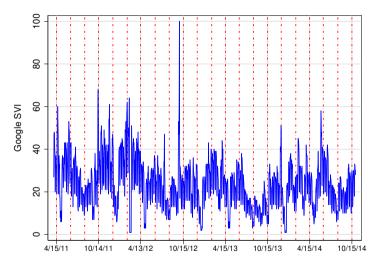




• Restrict to age \geq 65 y.o. and interact payday dummy with length of pay cycle p:

$$\frac{X_{i,c,t}}{\overline{X}_{i,c}} = \beta_1 \cdot \mathbb{1}\left(Payday_t\right) \times Length_{t \in p} + \beta_2 \cdot \mathbb{1}_i\left(Payday_t\right) \times \left(Length_{t \in p}\right)^2$$

- Control: $\mathbb{1}\Big(Payday_t\Big)=0 \implies C_0=\bar{c}$
 - $ar{c}$ not pinned down for pensioner pay cycles if include non-recipients in the control
 - ▶ Hence, we use $Payday_t$ rather than $Payment_{i,t}$ here
- Treatment: $\mathbb{1}\left(Payday_t\right) = 1 \implies C_0 = \bar{c} \cdot \left(1 + \underbrace{\beta_1 T + \beta_2 T^2}_{\equiv x(T)}\right)$
- $Length_{t \in p}$ varies between 57 and 62 days in our sample

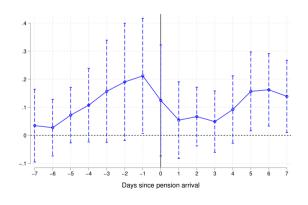


Notes: The figure displays the daily time series of the Japanese Google SVI for "public pension payments." Dashed red lines indicate scheduled pension payment dates during our sample period for the scanner data.

Consumers highly attentive to pension payment receipt



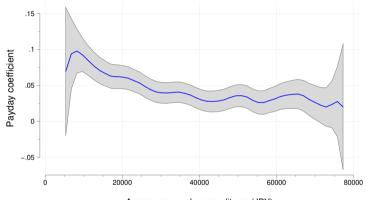
$$\widetilde{SVI_t} = \sum_{j=-7}^{+7} \beta_j \cdot Payday_{t+j} + \gamma \cdot t + \delta_{dow} + \phi_{wom} + \psi_{my} + \xi_h + \alpha_p + \epsilon_t$$



- Run time series regressions using Google SVI for "public pension payments" relative to average daily SVI as outcome
 - Include linear time trend and dummies α_p for other pension system announcements
- Search activity peaks (20% ↑) on the day prior to a scheduled payday
- Placebo w/randomized paydays shows no search spike inattention unlikely to play a role here

FLAT RELATIONSHIP BETWEEN PERMANENT INCOME AND PAYDAY RESPONSE

$$X_{c.t}^i/\overline{X}_{i.c}=eta^i\cdot Payday_t+\delta_{dow}+\phi_{wom}+\psi_{my}+\xi_h+\epsilon_t^i$$
 (Main deck)



Average pay cycle expenditures (JPY)

99% confidence interval — Payday response

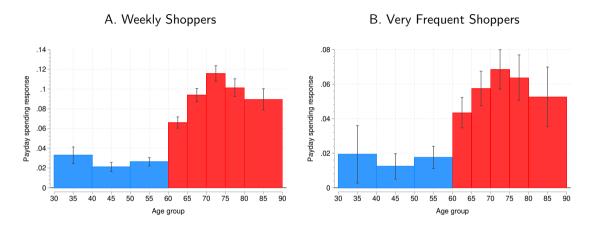
Notes: We estimate the time series regression pictured above for each individual shopper ID using all goods expenditures. The figure fits a local linear function to the relationship between payday responses $\widehat{\beta^i}$ and average expenditures over the two-month pay cycle. We winsorize the top 1% of daily expenditures. 99% confidence intervals represented by the gray shaded area.

Payday Expenditure Responses by Permanent Income Decile



	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Intensive margin	0.106***	0.097***	0.088***	0.074***	0.069***	0.054***	0.052***	0.048***	0.035***	0.031***
	(0.011)	(0.009)	(800.0)	(0.007)	(0.006)	(0.006)	(0.005)	(0.004)	(0.003)	(0.003)
Intensive margin w/controls	0.0104***	0.094***	0.085***	0.073***	0.067^{***}	0.051***	0.050***	0.046***	0.033***	0.029***
	(0.011)	(0.009)	(800.0)	(0.007)	(0.006)	(0.006)	(0.005)	(0.004)	(0.003)	(0.003)
Total response	0.069***	0.062***	0.047***	0.045***	0.036***	0.035***	0.026***	0.029***	0.022***	0.003
	(0.005)	(0.005)	(0.005)	(0.005)	(0.004)	(0.004)	(0.004)	(0.004)	(0.003)	(0.003)
Extensive margin	0.008***	0.007***	0.003*	0.003**	0.002	0.001	-0.002^{+}	-0.002^{+}	-0.002^{+}	-0.012***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)



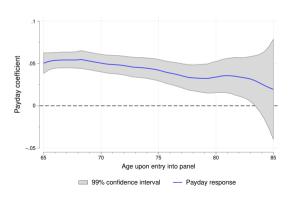


Notes: Point estimates relative to the 20-29 age group. Very frequent shoppers are those for whom we can match average total grocery spending with the FIES (i.e. those who visit a store, on average, at least 16 times per month). Capped bars indicate 99% confidence intervals with standard errors clsutered by shopper ID.

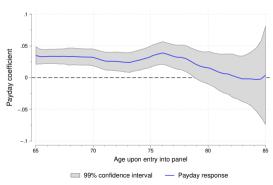
LITTLE DIFFERENCE IN RESPONSES BETWEEN YOUNGER VS. OLDER PENSIONERS

$$X_{i,t}/\overline{X}_i = \beta^i \cdot Payday_t + \delta_{dow} + \phi_{wom} + \psi_{my} + \xi_h + \epsilon_t^i \quad \text{Main deck}$$

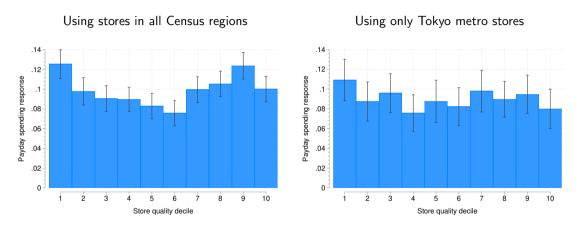




B. Very Frequent Shoppers



Payday spending response does not vary much with store quality



- Within-Tokyo analysis takes out CoL differences across areas
- Nearly identical point estimates if define store quality using equal-weighted vs. sale share-weighted (figure above) price index

STORE QUALITY MEASURE USING PRICES

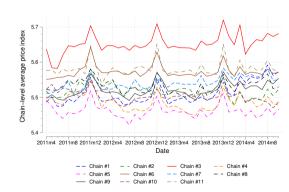


$$\widetilde{\Phi}_s = \frac{1}{|T^{np}|} \sum_{t \in T^{np}} \Phi_{s,t} = \frac{1}{|T^{np}|} \sum_{t \in T^{np}} \left(\sum_{k \in \Omega_{s,t}} \omega_{k,s,t} \log p_{k,s,t} \right)$$

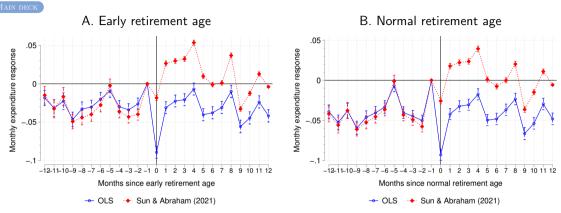
A. Sales Share-Weighted Retail Chain Index

- - Chain #11

B. Equal-Weighted Average Retail Chain Index



Age cohort heterogeneity in retirement consumption responses



- People cross retirement age threshold y^* at different times $\implies \mathbb{1}\{age_{i,t} \geq y^*\}$
- OLS shows drop in consumption, while other DiD estimators show uptick
 - ► Sun & Abraham (2021): compare retirees to never treated (young)
 - ▶ de Chaisemartin & D'Haultfœuille (2020): compare retirees to not-yet-retired (fuzzy RD)

FORMAL DECOMPOSITION OF STORE-LEVEL INFLATION



• Can write store-level inflation (geometric avg.) as the sum of three terms:

$$\Delta \Phi_{s,t} = \underbrace{\frac{1}{n_{s,t}} \sum_{k \in \Omega^*} \Delta \log p_{k,s,t}}_{\text{retail price change of common goods}} + \underbrace{\left(\frac{1}{n_{s,t}} - \frac{1}{n_{s,t-1}}\right) \sum_{k \in \Omega^*} \log p_{k,s,t-1}}_{\text{consumer variety response}}$$

$$+ \underbrace{\left(\frac{1}{n_{s,t}} \sum_{k \in \Omega^{new}} \log p_{k,s,t} - \frac{1}{n_{s,t-1}} \sum_{k \in \Omega^{old}} \log p_{k,s,t-1}\right)}_{(5)}$$

consumer substitution towards new goods

- Retail price change can be due to temporary sales or changes in regular prices
- Variety effect $\propto \Delta \log n_{s,t} \longrightarrow 6\%$ jump in # of unique barcodes purchased on payday
- ullet Last price index partitions Ω into a set of common, new, and old goods

ROBUSTNESS TO DEFINITION OF LAST PRICE INDEX



$$\Phi_{s,t} = \sum_{j=-7}^{+7} \gamma_{1,j} \cdot Payday_{t+j} + \delta_{dow} + \phi_{wom} + \xi_h + \eta_s + \varphi_{c,my} + \epsilon_{s,t}$$

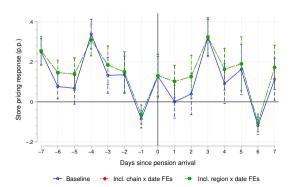
$$\Phi_{s,t}^{last} = \sum_{j=-7}^{+7} \gamma_{2,j} \cdot Payday_{t+j} + \delta_{dow} + \phi_{wom} + \xi_h + \eta_s + \varphi_{c,my} + \upsilon_{s,t}$$

A. Month Before Counterfactual Price

and a service person arrival

Incl. chain x date FEs - Incl. region x date FEs

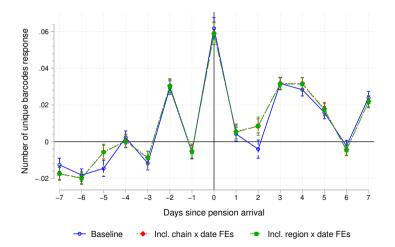
B. Week Before Counterfactual Price



Variety response: $6\% \uparrow$ in # of unique barcodes purchased on payday

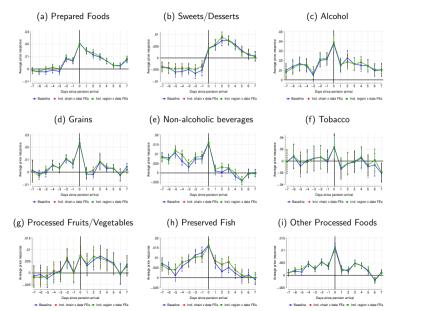
$$\log \widetilde{n}_{s,t} = \sum_{j=-7}^{+7} \gamma_{1,j} \cdot Payday_{t+j} + \delta_{dow} + \phi_{wom} + \xi_h + \eta_s + \varphi_{c,my} + \epsilon_{s,t}$$
 Main





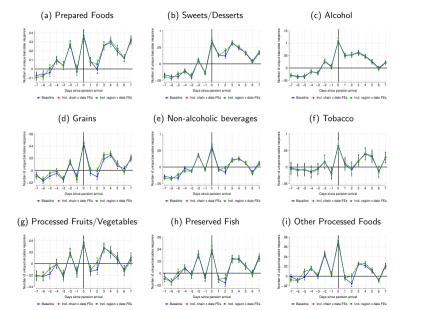
Response of Store-Level Major Subcategory Average Prices to Payday





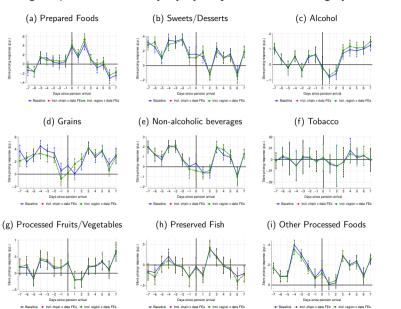
Store-Level Variety Responses within Each Major Goods Subcategory





Store Pricing Responses around Payday by Major Goods Subcategory





APPLYING TEMPORARY SALES FILTERS



- Apply two sets of filters to separate regular prices r_t from observed prices p_t :
 - Rolling mode (Kehoe & Midrigan 2008,15): regular price = most common price
 - $fence{2}$ V-shaped (Nakamura & Steinsson 2008): identify temporary sales by symmetric dips followed by rebounds ightarrow good out-of-sample prediction of sales flag in CPI microdata
- ullet Compute store-level average temporary sales frequency $\overline{f_s}$ and discount rate $\overline{d_s}$ via:

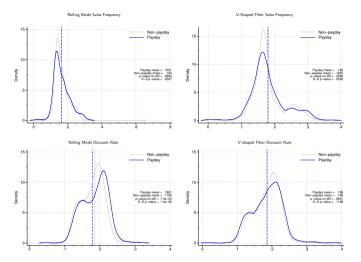
$$\overline{f_s} = \frac{1}{|T|} \sum_{t \in T} \left(\frac{1}{|K_s|} \sum_{k \in K_s} \mathbb{1}_t \{ p_{s,t,k} < r_{s,t,k} \} \right)$$

$$\overline{d_s} = \frac{1}{|T|} \sum_{t \in T} \left(\frac{1}{|K_s|} \sum_{k \in K_s} \left(1 - p_{s,t,k} / r_{s,t,k} \right) \right)$$

• Tuning parameters: search for V-shape and 3-month centered mode over 42 days (1.5 months), similar patterns if search over one week

Store-Level Temporary Sales Frequency and Discounts

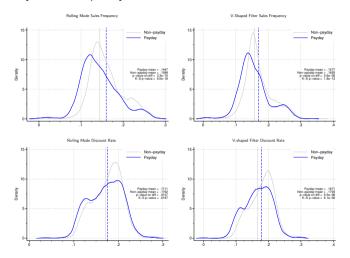




Notes: The left-hand side panels show the frequencies and discount rates under the rolling mode filter, while the right-hand side panels show the distributions when we use the V-shaped filter to identify sales. In both algorithms we search for temporary sales over a 42-day window on either side of a calendar date. Solid grey vertical lines indicate the mean daily frequency or discount rate across stores on non-paydays, while blue dashed lines show the mean across stores on paydays. The K-S p-value shows the two-sided exact p-value from a Kolmogorov-Smirnov test of equality for the payday vs. non-payday distributions.

Store-Level Temporary Sales Frequency and Discounts on Above-Median Price Goods

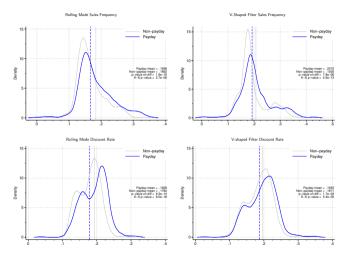




Notes: Includes only products which have an above-median average price within their four-digit goods category. The left-hand side panels show the frequencies and discount rates under the rolling mode filter, while the right-hand side panels show the distributions when we use the V-shaped filter to identify sales. In both algorithms we search for temporary sales over a 42-day window on either side of a calendar date. Solid grey vertical lines indicate the mean daily frequency or discount rate across stores on non-paydays, while blue dashed lines show the mean across stores on paydays. The K-S p-value shows the two-sided exact p-value from a Kolmogorov-Smirnov test of equality for the payday vs. non-payday distributions.

Store-Level Temporary Sales Frequency and Discounts on Below-Median Price Goods

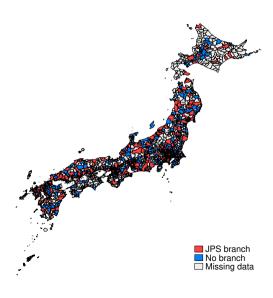




Notes: Includes only products which have a below-median average price within their four-digit goods category. The left-hand side panels show the frequencies and discount rates under the rolling mode filter, while the right-hand side panels show the distributions when we use the V-shaped filter to identify sales. In both algorithms we search for temporary sales over a 42-day window on either side of a calendar date. Solid grey vertical lines indicate the mean daily frequency or discount rate across stores on non-paydays, while blue dashed lines show the mean across stores on paydays. The K-S p-value shows the two-sided exact p-value from a Kolmogorov-Smirnov test of equality for the payday vs. non-payday distributions.

Statistics for Branch vs. Non-Branch Office Cities

	Branch	Non-branch	Difference	p-value
	(N = 239)	(N = 424)		
Log Census population	12.16	11.07	1.09	0.00
	(0.06)	(0.03)	(0.06)	
CBD population density ($1000s/km^2$)	7.66 (0.46)	5.11 (0.15)	2.55 (0.38)	0.00
Fraction population > 65 y.o. (%)	10.71	11.17	-0.46	0.09
	(0.17)	(0.18)	(0.26)	
Fraction population > 75 y.o. (%)	4.05	4.26	-0.21	0.07
	(0.07)	(0.08)	(0.12)	
Δ^{75-85} population $>$ 65 y.o.	43.00	46.73	-3.73	0.02
	(0.94)	(1.12)	(1.65)	
Fraction female residents (%)	51.14	51.09	0.05	0.63
	(0.00)	(0.00)	(0.00)	
Fertility rate	2.33	2.28	0.05	0.04
	(0.02)	(0.02)	(0.03)	
Log per capita income	7.82	7.79	0.03	0.01
	(0.01)	(0.01)	(0.01)	
Labor force participation rate (%)	50.24	49.71	0.53	0.06
	(0.21)	(0.16)	(0.28)	
Unemployment rate (%)	3.48	3.08	0.40	0.00
	(0.09)	(0.06)	(0.10)	
Ratio of govt. expenditures to revenues	0.97	0.97	0.00	0.89
	(0.00)	(0.00)	(0.00)	
Log welfare spending per person $>$ 65 y.o.	4.10	4.05	0.05	0.18
	(0.03)	(0.02)	(0.04)	
$\label{eq:log_log_log} \mbox{Log welfare spending per person} > 75 \mbox{ y.o.}$	5.08	5.03	0.05	0.19
	(0.03)	(0.02)	(0.04)	



Effect of Pension Frequency Reform on Municipal Admin Costs

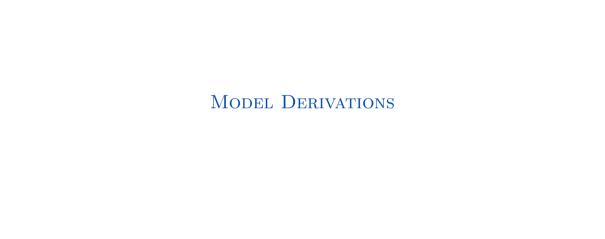
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	(1)	(2)	(3)	(4)	(5)	(6)
$Branch \times Post$	0.043**	-0.012	0.034	-0.003	0.073***	0.021
	(0.017)	(0.014)	(0.023)	(0.016)	(0.026)	(0.017)
City & year FEs	✓	✓	✓	~	✓	
Incl. Tokyo	~		✓		✓	
Incl. major cities	~		✓		✓	
1985 population bin $ imes$ year FEs			✓	✓	✓	✓
1985 per capita income bin \times year FEs					✓	✓
N	11,111	10,635	11,111	10,635	11,111	10,635
# Municipalities	663	635	663	635	663	635
Adj. \mathbb{R}^2	0.517	0.554	0.856	0.863	0.863	0.866

Notes: The dependent variable in each regression is log expenses on elderly welfare per resident at or above age 65. $Branch_j=1$ if municipality j contains a Japan Pension System branch office. $Post_t=1$ for years 1988-1996. All regressions include observations for years 1980-1996 and a full set of year fixed effects. Robust standard errors clustered at the municipality level in parentheses. Tokyo consists of the 23 central wards for which separate expenditure time series are available. Major cities consist of the historically five most populous cities outside of Tokyo: Yokohama, Nagoya, Kyoto, Osaka, and Kobe. 1985 population bin refers to quintiles of 1985 Census population. 1985 per capita income bin refers to quintiles of per taxpayer taxable income in 1985. ***p < 0.01, **p < 0.05, *p < 0.1



- Policy parameters (based on FY 2011 data from JPS)
 - Average daily payment per claimant: $\overline{B}=3,462$ JPY (\approx \$32.50)
 - Fraction of participants who receive benefits: p = 0.3766
- Administrative cost function
 - ▶ Posit cost function takes form $\mu(T) = \kappa_{\ell}/T^{\ell}$ where $\ell \geq 1$
 - For each ℓ calibrate scale factor κ_ℓ such that $\mu(T)$ matches reported administrative costs: 300.722 billion JPY (\approx \$3.07 billion)
- QH discounting
 - Set $f(t) = \nu \cdot t = 0.002t$ to match estimated 10% spike at payday
 - lacktriangle Avg. daily decline of 0.2% of consumption over pay cycle after stripping out seasonality
- Payday liquidity
 - ▶ Set magnitude of spike at x=0.1 or x(T)=0.0013T to match baseline estimates for raw foods (perishables \approx consumption)



- ullet Fraction p of people receive a flat (pension) benefit every T days equal to $b(T)=\overline{B}\cdot T$
- ullet Other 1-p fraction are workers who earn exogenous w(t) and pay lump-sum tax au(b)
- ullet Continuous time setup because T is the government's choice variable
- Government runs balanced budget for each $t \in [0, T]$:

$$(1-p)\cdot\tau(b) = p\cdot b(T) + \mu(T) \implies \tau(b) = \frac{p\cdot B\cdot T + \mu(T)}{1-p}$$

- \bullet $\mu(T)$ is an administrative cost function assumed to be weakly convex
 - ightharpoonup Captures program costs that vary with T: authorizing/delivering benefits, redeeming benefits, investigating fraud



- ullet Govt. picks T to minimize welfare loss subject to balanced budget
- Can write this compactly as

$$\min_{T} \left\{ -p \cdot \lambda(T) + \gamma \cdot \left(p \cdot b(T) + \mu(T) \right) \right\} \quad \text{with } \gamma = -\frac{\partial \mathcal{U}^* / \partial \tau}{\partial R^* / \partial \tau}$$

- ullet γ is the marginal cost of funds (MCF), equal to unity under lump-sum taxation of workers
- \bullet Govt. sets length of pay cycle T^* to equate marginal reduction in the welfare loss to marginal cost of reducing T

$$\underbrace{\frac{p \cdot \lambda'(T^*)}{\gamma}}_{\text{marginal benefit}} = \underbrace{\mu'(T^*) + p \cdot \overline{B}}_{\text{marginal cost}}$$
 (6)

 Sufficient statistics: fraction of pensioners, the average daily benefit amount, slope of welfare loss and cost function



• Working households face standard consumption-saving problem:

$$\max_{\{C(t)\}_{t\geq 0}} \int_0^T u(C(t)) dt \text{ s.t. } C(t) = S(t) + w(t) - \frac{\tau(b)}{T}$$

- Solution to this problem is full smoothing: $C(t) = C^*, \forall t$
- Optimal consumption for pensioners is also $C(t) = C^*, \forall t$, but suppose instead actual choice follows:

$$C(t) = c_0(T) \cdot \exp(-f(t))$$

- \bullet f(t) captures how path **deviates from optimum** over pay cycle
- Budget constraint $\int_0^T C(t)dt = b(T)$ pins down the value of consumption on payday $c_0(T)$ with f(0) = 0

• Welfare loss from non-smoothing is share λ willing to give up to achieve $C_t = C^*$

$$\int_0^T u^r \Big(c_0(T) \cdot \exp(-f(t)) \Big) dt = \int_0^T u^r (\lambda \overline{B}) dt$$

- \bullet $(1-\lambda)$ is the **compensating variation** or welfare loss from non-smoothing à la Lucas (1987), which depends on T
- For any invertible $u(\cdot)$ we can rewrite $\lambda(T)$ as

$$\lambda(T) = \frac{T \cdot u^{-1} \left\{ \frac{1}{T} \int_0^T u \left(c_0(T) \cdot \exp(-f(t)) \right) dt \right\}}{\int_0^T c_0(T) \cdot \exp(-f(t)) dt}$$

- Numerator: total consumption where daily consumption is s.t. receive average daily utility over the <u>actual</u> consumption path
- Denominator: actual total consumption over the pay cycle

Main deck

• Write compensating variation $\lambda(T)$ as a function of total observed consumption C^{tot} and total certainty equivalent consumption \overline{C} :

$$\lambda(T) = \frac{T \times \overline{C}}{C^{tot}} \implies \lambda'(T) = \frac{(T \times \overline{C})' \cdot C^{tot} - (T \times \overline{C}) \cdot (C^{tot})'}{(C^{tot})^2}$$

$$(C^{tot})' = \frac{\partial}{\partial T} \int_0^T c_0(T) \cdot \exp(-f(t)) = c_0 \cdot \exp(-f(T))$$

$$(\overline{C})' = \frac{\partial}{\partial T} u^{-1} \left\{ \underbrace{\frac{1}{T} \int_0^T u \Big(c_0(T) \cdot \exp(-f(t)) \Big) dt}_{\equiv \overline{U}(T)} \right\}$$

$$= u^{-1} \Big(\overline{U}(T) \Big) (u^{-1})' \Big(u(\overline{U}(T)) \Big) \cdot \underbrace{\frac{1}{T} \Big[u \Big(c_0(T) \cdot \exp(-f(T)) \Big) - \overline{U}(T) \Big]}_{=\overline{U}(T)}$$



Suppose utility function takes the form:

$$u(c_0) + \beta \cdot \sum_{t=1}^{T} \delta^t u(C_t)$$

- Individuals with these preferences exhibit present bias: sequence of discount rates is $1, \beta \delta, \beta \delta^2, \beta \delta^3, \ldots$, with $\beta < 1, \delta < 1$
- With $u(\cdot)$ isoelastic with inverse IES ρ , log consumption decreases over time

$$\frac{\partial \log(C_t)}{\partial t} = \frac{1}{\rho} \cdot \log \beta - \frac{1}{T - t + 1} + \frac{1}{T - t + \beta^{-1/\rho}} < 0$$

- For $\beta \approx 1$ but $\beta < 1$ this decrease is approximately linear
- Embed these preferences in the general model by assuming $f(t) = \nu t$ where ν is the **daily** decline in consumption over the pay cycle

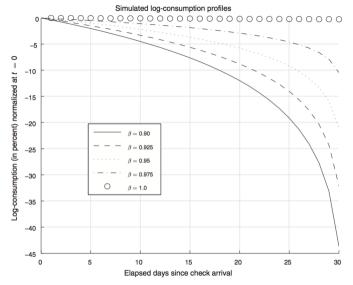


Figure 1. Monthly Consumption Pattern: $\delta=0.97; \, \rho=1$

Source: Mastrobuoni & Weinberg (2009)



• With $\overline{B} \cdot T$ to spend over the time period [0, T-1], budget constraint pins down payday consumption $c_0(T)$:

$$\int_0^T c_0(T) \cdot \exp(-\nu t) dt = \overline{B} \cdot T \implies c_0(T) = \frac{\nu \cdot \overline{B} \cdot T}{1 - \exp(-\nu T)}$$

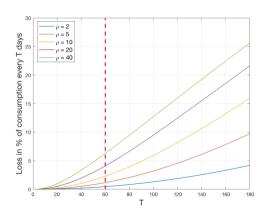
• Assuming isoelastic utility with inverse IES ρ the welfare loss is:

$$1 - \lambda(T) = \begin{cases} 1 - \frac{1}{\overline{B}} \cdot \exp\left(c_0 - \nu T/2\right) & \text{if } \rho = 1\\ 1 - \frac{c_0}{\overline{B}} \cdot \left[\frac{1 - \exp\left((\rho - 1)\nu T\right)}{\nu T(1 - \rho)}\right]^{\frac{1}{1 - \rho}} & \text{if } \rho \neq 1 \end{cases}$$

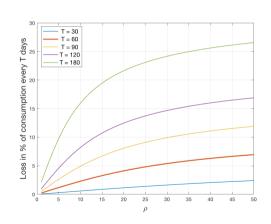
Welfare loss from QH discounting



By Interval Length



By Inverse IES



Intuition: Welfare loss from QH discounting



- With QH discounting, internality problem because individual overconsumes in earlier periods and underconsumes in later periods
- Three features of the welfare loss (1λ) :
 - lacktriangle Welfare loss is increasing in govt. choice of T
 - \star For higher T, welfare loss will be greater because integral between the optimal smooth path and QH path larger
 - **2** Welfare loss is increasing in ρ
 - \star Higher ρ means consumption less substitutable between periods, so individual willing to pay more to get closer to consumption smoothing
 - **3** Optimal T^* is decreasing in ρ
 - \star Govt.'s MB curve of decreasing T becomes steeper for higher ρ

CONTINUOUS QUASI-HYPERBOLIC DISCOUNTING



- Directly modeling quasi-hyperbolic discounting in continuous time is challenging because there is no clear "today" and "tomorrow"
- Following Webb (2016), define η as present period, and $0 \le \xi \le 1$ interval length:

$$V_{\eta}(C) = \underbrace{\int_{\eta}^{\eta+\xi} (\beta^{1/\xi}\delta)^{t-\eta} \cdot u(C(t)) dt}_{\text{instant gratification}} + \underbrace{\beta \int_{\eta+\xi}^{\infty} \delta^{t-\eta} u(C(t)) dt}_{\text{geometric discounting}}$$

- Utility at $t \eta = 0, 1, 2, \ldots$ weighted by $1, \beta \delta, \beta \delta^2, \ldots$
- Extra parameter ξ captures time interval before present bias kicks in
- Present bias in continuous time is akin to instant gratification
- Implied consumption path is similar to path in discrete time setting



- Recent papers find that individuals exhibit "payday liquidity"
 - ► Spike in expenditures on payday across the income distribution
 - Unrelated to expectations of future liquidity constraints
 - ► Expenditures are smooth for the rest of the pay cycle
- Simple consumption rule where t=0 is payday and interval T>1:

$$C_t = \begin{cases} (1+x) \cdot \bar{c} & \text{if } t = 0\\ \bar{c} & \text{if } t \in [1, T-1] \end{cases}$$

- If this C_t is the result of utility maximization then no welfare loss
- One possible utility function where this is optimal:

$$u(C) = (1+x) \cdot \log u(C_0) + \sum_{t=1}^{T-1} \log(C_t)$$

Welfare loss from myopic payday liquidity



- If underlying preferences do not put extra weight on $u(C_0)$ there will be a welfare loss
 - Can think of this as mental accounting under myopia
- ullet With convex administrative costs the welfare loss goes to zero as $T o \infty$
 - ► Trivial case where govt. faces no tradeoff
 - Intuition: loss is concentrated on the initial spike, so as $T \to \infty$ loss is small relative to total consumption over the cycle
- If instead allow spike to depend on pay cycle length with x'(T)>0, then $T^*<\infty$
 - ▶ Stephens & Unayama (2011): some evidence that x'(T)>0, since MPC out of pension payments lower when $T=90\to60$
 - ▶ We find evidence to support this case using high frequency data



- To keep things simple, focus on log utility ($\rho = 1$)
- Welfare loss expression is the same, but now the marginal loss is:

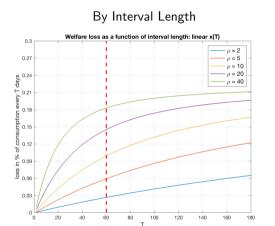
$$-\lambda'(T) = \frac{(\bar{c}/\overline{B})(1+x(T))^{1/T-1}}{T^2} \left[\left(1 + x(T) \right) \cdot \log \left(1 + x(T) \right) - Tx'(T) \right]$$

• Now decreasing T can improve welfare if:

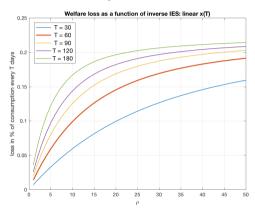
$$\underbrace{\left(1 + x(T)\right) \cdot \log\left(1 + x(T)\right)}_{\text{loss from spike magnitude as } T\uparrow} > \underbrace{T \cdot x'(T)}_{\text{gain from subdivision as } T\uparrow}$$
(7)

- ullet Spike grows with T due to pent-up demand (LHS), but <u>daily loss</u> falls as interval length increases (RHS)
- ullet Our empirical evidence suggests linear $x(T)=0.0023\cdot T \implies$ welfare higher for lower T

Welfare loss under payday liquidity with pent-up demand x(T)



By Inverse IES



- ullet $1-\lambda(T)$ is now concave vs. convex in the QH discounting case
- ullet Importantly, $\lambda'(T)$ has similar shape for both cases

Main deck

Extensions & Counterfactuals

Extension #1: sophisticated consumers with commitment



- Baseline framework assumes consumers fully naive about overspending around payday
 - ► Alternatively they could internalize temptation to spend earlier and adjust consumption to limit overspending by future self —> "sophistication"
 - ▶ If sophisticated might also want to commit to not overspending (Bryan, Karlan, Nelson 2010)
- Forms of commitment devices: layaway, retirement/education savings accounts, timing services payments (e.g. utilities, mortgage) to coincide with income
- Idea: more infrequent payments makes it easier for consumers to save up for large durable purchases like appliances (Zhang 2023)
- **Key result:** allowing for commitment via durables purchases leads to $T^* \uparrow$, but not by much unless IES is very low (ρ is very high)
 - ightharpoonup IES determines preference for commitment and lower T makes it harder to commit

Modeling durable commitments: a nested agent problem



- \bullet Simulate for range of ρ and T consumption path for three types of consumers who overspend on payday
 - ightharpoonup Parameterize as present-bias problem, but can map back to mental accounting via u
- lacktriangledark Naive: consumers in our baseline model who choose consumption plan $\{c_t^*\}_{t=0}^{T-1}$
- ② Sophisticated: are aware of present-bias and solve for the optimal consumption plan via backwards induction to obtain $\{c_t^{**}\}_{t=0}^{T-1}$
 - lacktriangle Algorithm: solve naive problem and iterate backwards to obtain c_0^{**}
- **Sophisticated** + commitment (SC): given access to a commitment device z_0 which allows withholding on payday and (linearly) amortizing in future periods
 - Give up z_0 initially to gain $z_0/T-1$ in future when consumption is below the smooth level due to overspending on payday
 - ▶ Linear subdivision proxies for economic depreciation of durables



• Sophistication + commitment problem collapses to:

$$\max_{z_0} \left\{ u(c_0^{**} - z_0) + \beta \sum_{t=1}^{T-1} \delta^t u(c_t^{**} + z_0/(T-1)) \right\} \text{ s.t. } \begin{cases} z_0 \ge 0 \\ c_0^{**} - z_0 > 0 \end{cases}$$
 (8)

- For log utility $(\rho = 1)$ well-known result that no preference for commitment, so $z_0^* = 0$, meaning the slackness or "no borrowing" constraint binds
- Continuous time approximation of c_t^{**} is then

$$C(t) = \exp\left(\theta - f(t) + \zeta(t)\right) \tag{9}$$

- Cumulative "pull-back" towards the optimum $Z(T)=\int_0^T \zeta(t)dt$ has the property $Z'(T)\geq 0$ \longrightarrow it becomes more difficult to commit with shorter pay cycles
 - $\zeta(t)$ can be non-monotonic in t, depending on T and ρ

EXTENSION #2: MENU COSTS AND RETAILER RESPONSES



- ullet Baseline model assumes price P of consumption bundle does not vary w.r.t. T
 - Consistent w/empirical findings of minimal retailer pricing response when averaged over entire basket of commonly purchased goods
- Consider two extensions with monopolistic retailers:
 - lacktriangle Single monopolistic chain providing the entire basket C at price P
 - Continuum of monopolistic chains specializing in varieties
- ullet Retailers face fixed real inventory cost Γ and menu cost for changing prices
 - lacktriangleright Both versions of model illustrate that allowing for fixed cost of changing prices $\implies T^* \downarrow$
 - lacktriangle baseline calibration results provide upper bound on T^*



Lemma (neutrality result)

Consider optimal frequency model with single monopolistic chain that price discriminates on the extensive margin (i.e. P changes only on the time dimension, but not by demographics) and pays menu costs in units of wage labor.

Then there exists a set of parameters (p, A, κ, ω) s.t. for any T, consumer welfare the same regardless of whether there is price discrimination.

- Intuition: loss in utility over consumption associated with the price hike is completely offset by the reduction in disutility from labor supply
- Relies on idea that menu costs are mostly labor costs (Blinder et al. 1998)



• Retailer sets sequence of prices P_t to maximize profits:

$$\max_{\{P_t\}} \left\{ \sum_{t=0}^{T-1} P_t \cdot Y_t - W_t \cdot L_t - \kappa \cdot W_t \times \mathbb{1}_t - P_t \cdot \Gamma \right\}$$

- ullet Labor L_t used to satisfy non-recipient demand $F(L_t)=C_t^{NR}$
- Menu cost $\kappa \cdot W_t$ units of wages paid if change regular price ($\mathbb{1}_t = 1$)
- Non-recipients work and incur disutility from providing labor to the retailer:

$$\max_{\{C_t, L_t\}} \left\{ \sum_{t=0}^{T-1} u(C_t) - \nu(L_t) \right\} \quad \text{s.t.} \quad P_t \cdot C_t = S_t + W_t \cdot L_t - \frac{\tau(b)}{T}$$



$$\min_{T} \left\{ \underbrace{-p \cdot \lambda(T) + \gamma \cdot \left(p \cdot b(T) + \mu(T)\right)}_{\text{welfare loss from non-smoothing} + \text{ taxes}} + \underbrace{\left(U^*(C^{1,NR}(T)) - U^*(C^{0,NR}(T))\right)}_{\text{welfare loss from price discrimination}} \right\}$$

- If T^* s.t. retailer finds it unprofitable to price discriminate, $\mathbb{1}_0=0$, and non-recipients experience no utility loss
- ullet If instead admin costs $\mu(T)$ are sufficiently convex, then govt. may set T^* s.t. price discrimination occurs in equilibrium
- Parameterization: suppose payday liquid benefit recipients, non-recipients have $u(C_t) \nu(L_t) = \log(C_t) \omega \cdot L_t$, and production is linear in labor $F(L_t) = A \cdot L_t$



• Combine FOCs from the non-recipient's problem to get profits over the pay cycle:

$$\sum_{t=0}^{T-1} C_t - \omega C_t^{NR} \cdot \frac{C_t^{NR}}{A} - \kappa \cdot \omega C_t^{NR} \times \mathbb{1}_t - \Gamma$$

• Aggregate demand is the sum of the recipient (R) and non-recipient (NR) demands:

$$C_t = p \cdot C_t^R + (1 - p) \cdot C_t^{NR}$$

• Equilibrium real expenditures of non-recipients:

$$C_t^{NR} = \begin{cases} \frac{A}{2\omega} & \text{if } \mathbb{1}_t = 0\\ \frac{A\Big((1-p) - \kappa \cdot \omega\Big)}{2\omega \cdot (1-p)} & \text{if } \mathbb{1}_t = 1 \end{cases}$$

$$P_t = \begin{cases} & \frac{2W_t}{A} \text{ if } \mathbbm{1}_t = 0\\ & \frac{2W_t \cdot (1-p)}{A \cdot (1-p-\kappa \omega)} \text{ if } \mathbbm{1}_t = 1 \end{cases}$$

- Payday liquid recipients \implies price discrimination if it does occur will only be profitable on payday $\implies P_t = 2W_t/A$ for $t \neq 0$
 - ▶ Logic: C_t^R is smooth except for t=0 when people decide to splurge
 - Assume $\kappa \cdot \omega < (1-p)$ since $P_t > 0$ (disutility from labor or menu costs cannot be too large)
- Comparing profit functions, price discrimination occurs if and only if:

$$x(T) > \frac{1-p}{p\bar{c} \cdot \left((2-A)(1-p) - A \cdot \kappa \omega \right)} \cdot \left\{ A\kappa + \frac{A(1-p-\kappa \omega)}{(1-p)} \cdot \left(p\bar{c} + \frac{A(1-p) - \kappa \omega}{2\omega} \right) \right\}$$



$$x(T) > \frac{1-p}{p\bar{c} \cdot \left((2-A)(1-p) - A \cdot \kappa \omega \right)} \cdot \left\{ A\kappa + \frac{A(1-p-\kappa \omega)}{(1-p)} \cdot \left(p\bar{c} + \frac{A(1-p) - \kappa \omega}{2\omega} \right) \right\}$$

- ullet Gain from price discrimination = excess demand from price-inelastic pensioners receiving income o quantified by spike x(T) in the data
- Loss from price discrimination = menu cost + reduced demand from non-recipients
- Setting $T \downarrow \implies x(T) \downarrow$ and menu cost becomes a larger fraction of profits
- Difference in non-recipients' optimized level of utility when there is price discrimination is:

$$\log(C_0^{NR,1}) - \log(C_0^{NR,0}) + \omega \cdot \left(L_0^1 - L_0^0\right) = 0$$

ullet Punchline: price discrimination is welfare neutral \Longrightarrow govt. can ignore retailer!

Details: comparing costs under frequency vs. NRA reforms



Counterfactual exercise

What would be the increase in pay cycle length $\Delta T>0$ required to reduce costs by an equivalent amount to raising the NRA from 65 to 70?

- Calculate increase in penalty rates imposed on early pensioners aged 60-69 compared to current NRA of 65
 - ► Current penalty: 0.4-0.5% per month until the month of 65th birthday, capped at 30%
 - ▶ Under shift in NRA to 70, overall penalty would max out at 60%
- Assume claiming rate of 10.8% persists for 65-69 age group...
 - $\blacktriangleright \implies$ fraction of eligibles p decreases from 0.377 to 0.307
- Cost savings are lower if retain 30% penalty cap: 28.07 billion JPY vs. 36.12 billion JPY



- ullet Result: raising retirement age pushes up the optimal pay cycle length T^*
- ullet Raising NRA results in $p^{new} < p^{old} \implies$

$$\frac{1}{p^{new}} \cdot \mu'(T^*) = \frac{\kappa_{\ell}}{p^{new}} \cdot \left(-\ell/T^{*\ell-1}\right) < \frac{1}{p} \cdot \mu'(T^*)$$

• $\lambda'(T) < 0$, so as $p \downarrow$, MC of increasing frequency dominates the MB (welfare gains) from consumption smoothing

Admin cost	$T^*(p^{new})$	$T^*(p^{old})$		
convexity	$I^{-}(p^{-})$			
$\ell = 1$	7.55 days	6.72 days		
$\ell = 2$	19.63 days	18.27 days		
$\ell = 3$	28.81 days	27.32 days		