Public Economics: Lecture 13
Optimal Commodity Taxation

Cameron LaPoint

Columbia University

August 1, 2017

Optimal taxation theory

- Government's problem: given a revenue target R, how should taxation be structured in order to maximize welfare?
- Equivalently can think of maximizing welfare as minimizing the excess burden from taxes
- 1st best solution: impose lump-sum taxes on consumers that vary by ability to pay – no distortion in relative prices
- Lump-sum taxation based on redistribution of initial endowments not feasible in practice (Second Welfare Theorem fails)
- Optimal taxation theory explores the government's 2nd best strategy for raising R

Optimal commodity taxation: big picture

- Assumption: direct lump-sum taxation is not feasible, and there is at least one good that cannot be taxed (numeraire good)
- How should the tax base be distributed across the taxable goods?
- Intuition for the solution: govt. should rely more heavily on taxes that generate a relatively low excess burden per dollar of revenue
- From the efficiency cost lecture earlier in the course we already know the solution will feature...
  1. Inverse elasticity rule: smaller excess burden from taxing goods that are more demand inelastic
  2. Broad tax base rule: more efficient to spread taxes across all goods to keep tax rates low (marginal DWL is increasing in the tax rate)

Model setup

- Individuals have utility that is *quasi-linear* in a tax exempt good C_3:

$$u_1(C_1) + u_2(C_2) + C_3$$

- They receive income y and face the budget:

$$(p_1 + t_1) \cdot C_1 + (p_2 + t_2) \cdot C_2 + C_3 = y$$

- Government wants to raise revenue R from taxes imposed on goods C_1 and C_2, and thus faces the revenue constraint:

$$t_1 \cdot C_1 + t_2 \cdot C_2 = R$$

- Many possible combinations of t_1 and t_2 can raise R, but each has different welfare implications

Government's problem

- The government is utilitarian, so its social welfare function is simply the utility of the representative individual
- Government picks t_1 and t_2 to maximize taxpayer utility subject to two constraints:
  1. Individual optimization: given the government's choice of t_1 and t_2, need to take into account that individual taxpayers will optimize by picking C_1^*, C_2^*, C_3^*
  2. Revenue constraint: $t_1 \cdot C_1 + t_2 \cdot C_2 = R$
- We are agnostic about what the government chooses to do with collected revenue – no redistribution motive here since all taxpayers are assumed to be the same

Taxpayer's problem

- Quasi-linear utility allows us to substitute the budget constraint into the utility function for C_3 to get the maximization problem:

$$\max_{(C_1,C_2)} \left\{ u_1(C_1) + u_2(C_2) + y - (p_1 + t_1) \cdot C_1 - (p_2 + t_2) \cdot C_2 \right\}$$

- Solutions to this problem satisfy the two FOCs:

$$MU_1 = u_1'(C_1^*) = p_1 + t_1 \qquad MU_2 = u_2'(C_2^*) = p_2 + t_2$$

- Can view the solution to the taxpayer's problem as C_1^* and C_2^* satisfy the tangency conditions relative to the tax-exempt good C_3
- To simplify notation throughout we write $C_1^* \equiv C_1^*(p_1 + t_1)$ and $C_2^* \equiv C_2^*(p_2 + t_2)$ to denote the optimal consumption choice
- But important to remember that these choices depend on the prices/taxes in place!

Solving the government's problem

- Given the taxpayer's optimal choices C_1^*, C_2^* we can write the government's problem as

$$\max_{(t_1,t_2)} \left\{ u_1(C_1^*) + u_2(C_2^*) + y - (p_1 + t_1) \cdot C_1^* - (p_2 + t_2) \cdot C_2^* \right\}$$
$$\text{s.t. } t_1 \cdot C_1^* + t_2 \cdot C_2^* = R$$

- To solve this we will use the Lagrange multiplier approach:

$$\max_{(t_1,t_2)} \left\{ u_1(C_1^*) + u_2(C_2^*) + y - (p_1 + t_1)C_1^* - (p_2 + t_2)C_2^* + \lambda \cdot (t_1 C_1^* + t_2 C_2^*) \right\}$$

- Advantage to this approach: can interpret this problem as government maximizes a weighted sum of utility and revenue
- We will see that λ parameterizes the trade-off between raising one more dollar of revenue and the welfare cost of raising that dollar

FOCs of the government's problem

- Call the maximized welfare of the individual W^*
- Then for shorthand we can write the government's problem as

$$\max_{(t_1,t_2)} \left\{ W^* + \lambda \cdot (t_1 C_1^* + t_2 C_2^*) \right\}$$

- The two FOCs of the government's problem set the *total* marginal effect of a tax increase on each good equal to zero:

$$\frac{\partial W^*}{\partial t_1} + \lambda \cdot \left(C_1^* + t_1 \cdot \frac{\partial C_1^*}{\partial t_1} \right) = 0$$
$$\frac{\partial W^*}{\partial t_2} + \lambda \cdot \left(C_2^* + t_2 \cdot \frac{\partial C_2^*}{\partial t_2} \right) = 0$$

- Note that the taxpayer's choice C_1^* does not depend on t_2, and C_2^* does not depend on t_1 due to the utility function we assumed
  - $\implies$ Cross-price elasticities of demand are zero!

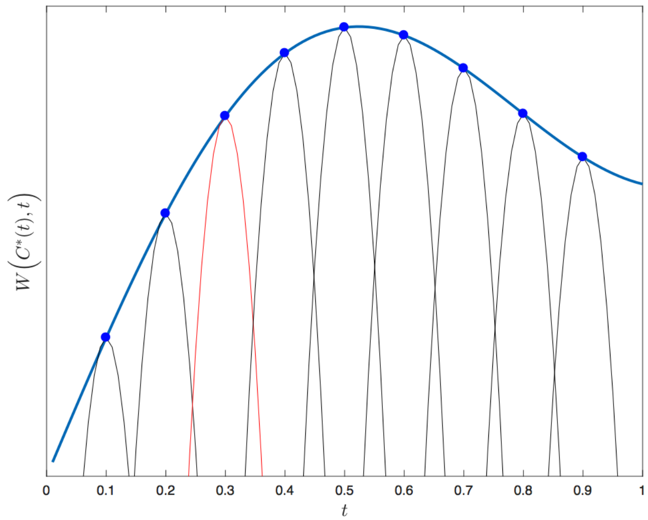
Envelope theorem

- How do we evaluate the effects of tax changes on the maximized welfare of the individual?
- Consider $\partial W^*/\partial t_1$:

$$-C_1^* + \frac{\partial}{\partial C_1}\left\{u_1(C_1^*) + u_2(C_2^*) + y - (p_1 + t_1)C_1^* - (p_2 + t_2)C_2^*\right\} \cdot \frac{\partial C_1}{\partial t_1}$$

- Due to the Envelope theorem, the second term above is equal to zero
- Intuition: taxpayer chooses optimal consumption as a function of each possible tax rate set by the government
  - $\implies$ slope in the C_1^* direction at the maximized level of welfare must be equal to zero
  - Hence, the effect of a change in a tax rate only has an effect on maximized welfare by altering the set of feasible choices

Envelope theorem – graphical intuition



- Since C^* is a function of t, the tax changes"envelope" the taxpayer's maximized level of welfare

Marginal cost of funds (MCF)

- Applying the Envelope theorem, the two FOCs of the government's problem reduce to:

$$-C_1^* + \lambda \cdot \left(C_1^* + t_1 \cdot \frac{\partial C_1^*}{\partial t_1} \right) = 0 \qquad -C_2^* + \lambda \cdot \left(C_2^* + t_2 \cdot \frac{\partial C_2^*}{\partial t_2} \right) = 0$$

- Rearranging the two FOCs we find that at the optimum:

$$\frac{C_1^*}{C_1^* + t_1 \cdot \frac{\partial C_1^*}{\partial t_1}} = \lambda = \frac{C_2^*}{C_2^* + t_2 \cdot \frac{\partial C_2^*}{\partial t_2}}$$

- This characterization of the optimum shows us that the λ can be interpreted as the marginal cost of funds (MCF)
  - MCF is the ratio of the effect of a small tax change on welfare relative to the effect of a small tax change on revenue
  - At the optimum, the MCF must be equalized across the two tax instruments t_1 and t_2

Review: price elasticity of demand

- We can rewrite the solution to the optimal commodity tax problem in terms of the price elasticity of demand for the two consumption goods
- The elasticity of X with respect to Y tells us the percent change in X when Y changes by 1%
- The price elasticity of demand for any good is:

$$\eta = \frac{\Delta C / C}{\Delta p / p} = \frac{\Delta C}{\Delta p} \cdot \frac{p}{C}$$

- For very small changes in the demand and prices we can write this elasticity as

$$\eta = \frac{\partial C}{\partial p} \cdot \frac{p}{C}$$

- In what follows, denote η_1 the demand elasticity of good 1 and η_2 the demand elasticity of good 2

Inverse elasticity rule

- First rearrange the optimality condition:

$$\frac{1}{1 + t_1 \cdot \frac{\partial C_1^*}{\partial t_1} / C_1^*} = \frac{1}{1 + t_2 \cdot \frac{\partial C_2^*}{\partial t_2} / C_2^*} \implies t_1 \cdot \frac{\partial C_1^*}{\partial t_1} / C_1^* = t_2 \cdot \frac{\partial C_2^*}{\partial t_2} / C_2^*$$

- Define price elasticity of demand with respect to the price:

$$\eta_i = \frac{\partial C_i^*}{\partial t_i} \cdot \frac{p_i + t_i}{C_i^*}, \text{ for } i = 1, 2$$

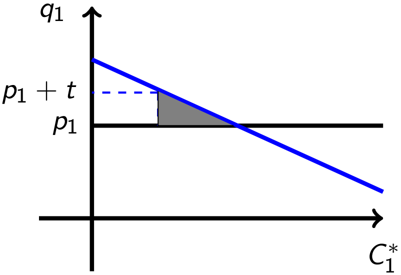
- Using this definition we have optimal commodity taxes satisfy:

$$\frac{t_1}{p_1 + t_1}\eta_1 = \frac{t_2}{p_2 + t_2}\eta_2$$

- Define the *ad valorem* tax rate $\tau_i \equiv \frac{t_i}{p_i + t_i}$
- The optimal condition thus reduces to an inverse elasticity rule:

$$\frac{\tau_1}{\tau_2} = \frac{\eta_2}{\eta_1}$$

Inverse elasticity rule – graphical intuition



- Good 1 more demand elastic than good 2 $\implies$ for the same tax t excess burden is larger for good 1 than good 2
- To minimize total excess burden, need to lower the tax rate on good 1 relative to the tax rate on good 2

Implications of this model

- Optimal commodity tax rule does not feature uniform taxation (i.e. the tax rates on the two goods will differ whenever $\eta_1 \neq \eta_2$)
- Exemption of goods from taxation difficult to justify – unless one is perfectly (in)elastic
- Empirical values for the elasticities may be used to determine the optimal structure of taxation implied by this model
- Again, we made two key assumptions that simplified the problem:
  - The demand for one good is unrelated to the other due to the separability of the utility function in C_1 and C_2
  - In general, there would be cross-price effects due to the substitutability/complementarity of the goods
  - No redistribution motive: there could be an equity-efficiency trade-off between taxing inelastic inferior goods and elastic luxury goods

Summary

- Standard optimal commodity tax theory presents government trade-off between raising required level of revenue and minimizing total excess burden from taxing multiple goods
- Importantly, no redistribution in this model, because all individuals are the same (i.e. one representative taxpayer)
- Result is the inverse elasticity rule: a good's tax rate is inversely related to its price elasticity of demand
- The rule is criticized as *regressive* tax policy since demand inelastic goods (e.g. food) form a larger fraction of expenditures for the poor