Public Economics: Lecture 1
Theoretical Tools & Consumption/Leisure Choice

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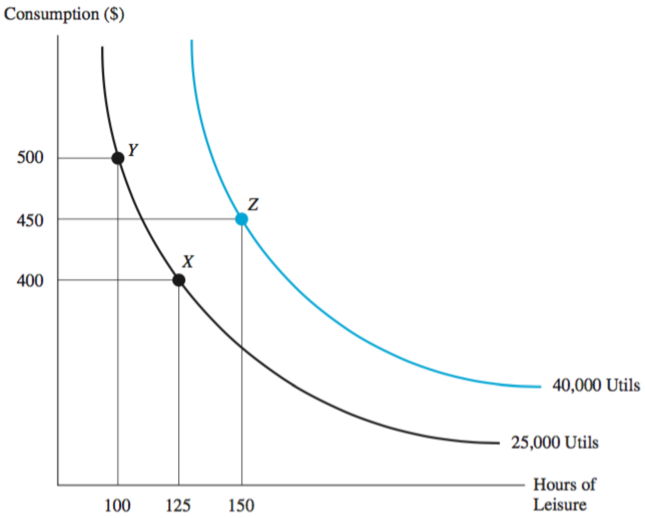
Utility functions

- A utility function is a mathematical representation of preferences over bundles of goods
- In this lecture we are primarily interested in bundles of consumption C and leisure L
- Commonly used utility functions:
  - $U(C, L) = C^{\alpha} L^{1-\alpha}$, with $0 \leq \alpha \leq 1$ (Cobb-Douglas utility)
  - $U(C, L) = a \log(C) + b \log(L)$ for constants $a > 0, b > 0$
- If I have two bundles (C, L) and (C', L') and $U(C, L) > U(C', L')$, I prefer the consumption/leisure choice represented by (C, L)
- Assumptions: individuals have well-defined preferences and make choices to achieve the highest possible utility level

Indifference curves

- Indifference curves are a graphical representation of preferences over bundles of goods
- Each curve traces out the set of bundles of C and L that give you the same fixed level of utility $\bar{U}$
- Individuals always prefer to pick bundles on higher indifference curves (utility maximization)
- The curves are always downward sloping – consuming more means you have to forgo more leisure to earn income
- Since we will always assume utility functions are concave, the indifference curves will always be convex

Consumption/leisure indifference curves



Source: G. Borjas, *Labor Economics*, Figure 2.2

Marginal rate of substitution (MRS)

- The slope of the indifference curve is the MRS – it is the ratio of the marginal utilities obtained from the two goods
- Example: $U(C, L) = C^{1/3}L^{2/3}$
- When the two goods are consumption (y-axis) and leisure (x-axis) the MRS can be computed as:

$$MRS = -\frac{MU_L}{MU_C} = -\frac{(2/3)C^{1/3} \cdot L^{-1/3}}{(1/3)C^{-2/3} \cdot L^{2/3}} = -\frac{2C}{L}$$

- This is the standard "rise over run" formula
  - The change in the y direction is the extra utility I get from a little more leisure
  - The change in the x direction is extra utility I lose from forgoing a little more consumption

Budget constraints

- Budget constraints define the set of feasible choices
- Typical intermediate micro budget constraint for two goods A and B with prices p_A and p_B and income Y:

$$p_A \cdot A + p_B \cdot B = Y$$

- As a general rule, the budget constraint equates total expenditure to total disposable income
- Disposable income means income left over after all taxes/transfers have been collected and distributed
- The constraints will look different in this course because we care about how taxes affect consumption and labor supply choice

Constrained optimization

- Consumption/leisure choice problems have the form:

$$\max_{C,L} \left\{ U(C, L) \right\} \text{ s.t. } \textit{budget constraint}$$

- The budget constraint depends on what we assume about the individual's ability to save, earn income, and taxes/transfers

- In a simple case with no savings and taxes and income from labor earnings, the budget is $C = w(1 - L)$
  - $1 - L$ is time spent working, with $0 \leq L \leq 1$ the fraction of time spent on leisure
  - w is the wage rate, or how much would be earned if all time were devoted to work ($L = 1$)
  - Price of consumption is normalized to 1 (C is nominal)

Solving the optimization problem

For any problem with goods A and B two conditions must be satisfied at the optimum:

1. The slope of the budget constraint and the indifference curve must be the same (tangency):

$$-\frac{MU_A}{MU_B} = MRS = -\frac{p_A}{p_B}$$

   Equivalently, the MRS must equal the relative price ratio

2. The optimum must lie on the budget constraint

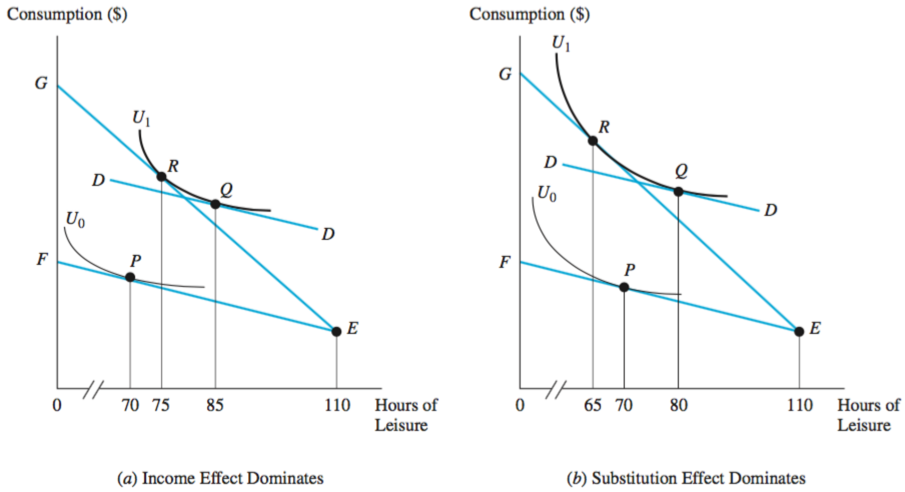
$$p_A \cdot A + p_B \cdot B = Y$$

   Otherwise there are unused resources that could be used to increase the level of utility

Income and substitution effects

- Income effects occur when the buying power of income changes
  - If buying power increases (decreases), people have incentive to consume more (less) of each good
- Substitution effects occur when the relative price of two goods changes
  - After a price change individuals have incentive to substitute towards goods that become relatively inexpensive
- When there is a change in relative prices, income effects and substitution effects occur simultaneously
  - Need to keep track of both types of effects on demand for both goods
  - Which effect dominates depends on the assumed utility function

What could happen when the wage rate increases?



Source: G. Borjas, *Labor Economics*, Figure 2.9

Adding in taxes and transfers

There are two basic kinds of taxes we could impose on labor income

1. Lump-sum tax (*non-distortionary*): the government collects a fixed amount of income T from the individual
   - T does not change relative prices and enters additively into the budget constraint: $C = w(1 - L) + T$
   - $T > 0$ is a lump-sum transfer, $T < 0$ is a lump-sum tax
2. Proportional tax (*distortionary*): the government collects a proportion t of income from the individual
   - t changes the relative price of consumption to leisure because the *after-tax* wage rate is the opportunity cost of leisure: $C = (1 - t)w(1 - L)$
   - $t < 0$ is a labor subsidy, $t > 0$ is a labor income tax

Non-linear budget constraints

- Many tax schemes and welfare programs introduce non-linearities such as kinks to budget constraints
  - Examples: Earned Income Tax Credit (EITC), tax exemptions, health insurance subsidies, Social Security earnings test

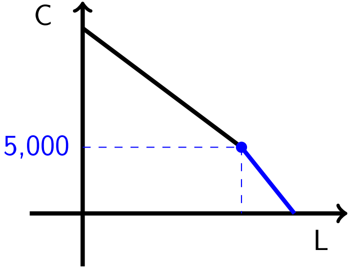
- What is the point of introducing a kink?
  - Redistribution: don't tax those who are very needy (tax exemptions) or tax those who are not so needy (earnings tests)
  - Incentivize labor supply among those who are able to work

- At a kink in a budget constraint, the relative price of consumption to leisure changes
- Need to solve separately for the optimal choices on budget segments before and after the kink and check feasibility of each solution

Tax exemption

- Suppose there is a tax that applies only when earnings $Y = w(1-L)$ exceed \$5,000
- The budget constraint now has two segments:

$$C = \begin{cases} w(1-L) & \text{if } Y \leq 5000 \\ \underbrace{w(1-L)}_{\text{earnings}} - \underbrace{t\Big[w(1-L) - 5000\Big]}_{\text{tax collected on earnings above exemption}} & \text{if } Y > 5000 \end{cases}$$



Elasticity

- The elasticity of X with respect to Y tells us the percent change in X when Y changes by 1%

$$\epsilon_{x,y} = \frac{\text{\% change in X}}{\text{\% change in Y}}$$

- We can compute the elasticity between any two variables, but in this course we are mostly interested in a few key elasticities:
  - The elasticity of supply or demand for a good with respect to the price of that good
  - Labor supply elasticity with respect to the generosity of a public transfer or an earnings tax rate
  - Elasticity of job search effort with respect to unemployment benefits (more on the economic significance of this later)

Theoretical elasticity

- Suppose we want to compute the price elasticity of demand
- In theoretical applications we take as given that we know the demand curve in its entirety
- $D(p)$ is quantity demanded of a good at a given price p
- Given that we know the functional form of the demand curve, we can compute the price elasticity via:

$$\epsilon = \frac{\Delta D(p)/D(p)}{\Delta p/p} = \frac{p}{D(p)}D'(p) \approx \frac{\Delta \log(D(p))}{\Delta \log(p)}$$

- The log approximation holds for small percentage changes to $D(p)$ and p: $\Delta \log(x) \approx \%\Delta x$, for any variable x

Empirical elasticity

- In reality we may not know the shape of the entire demand curve
- Instead we observe a limited set of data on quantities purchased and prices at different points in time
- Example: $p_0 = \$10$, $p_1 = \$12$; $D_0 = 200$, $D_1 = 180$
- We will get a different elasticity estimate depending on whether we use period 0 or period 1 as the initial period
- To avoid this issue, we take the *midpoint* of the values in periods 0 and 1 for prices and quantity demand:

$$\epsilon = \frac{\Delta D/[(1/2)(D_0 + D_1)]}{\Delta p/[(1/2)(p_0 + p_1)]} = \frac{(180 - 200)/(200 + 180)}{(12 - 10)/(10 + 12)} = -0.58$$

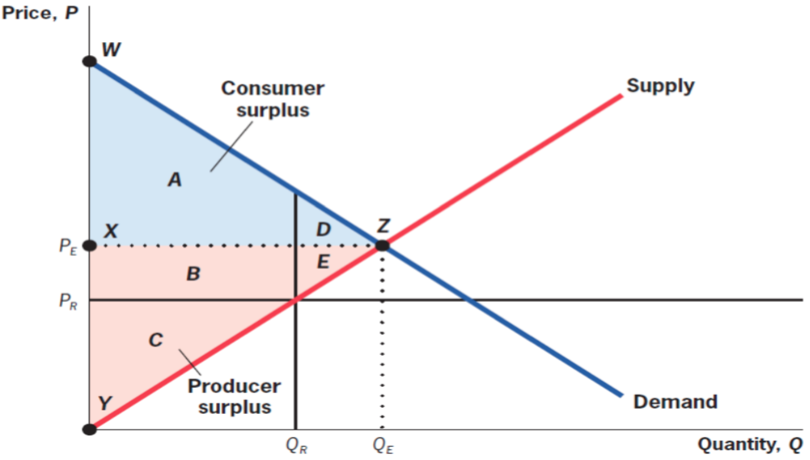
Elasticity – special cases

- Perfectly inelastic or vertical demand (supply) curve: $\epsilon = 0$ at all points – at any given price the quantity is constant
- Perfectly elastic or horizontal demand (supply) curve: $\epsilon = \infty$ at all points – at any given quantity the price is constant
- Elastic goods have $|\epsilon| > 1$
  - Examples: restaurant meals, computers, air travel, jewelry
- Inelastic goods have $|\epsilon| < 1$
  - Examples: gasoline, alcohol, appliances, cigarettes, housing

Market equilibrium and efficiency

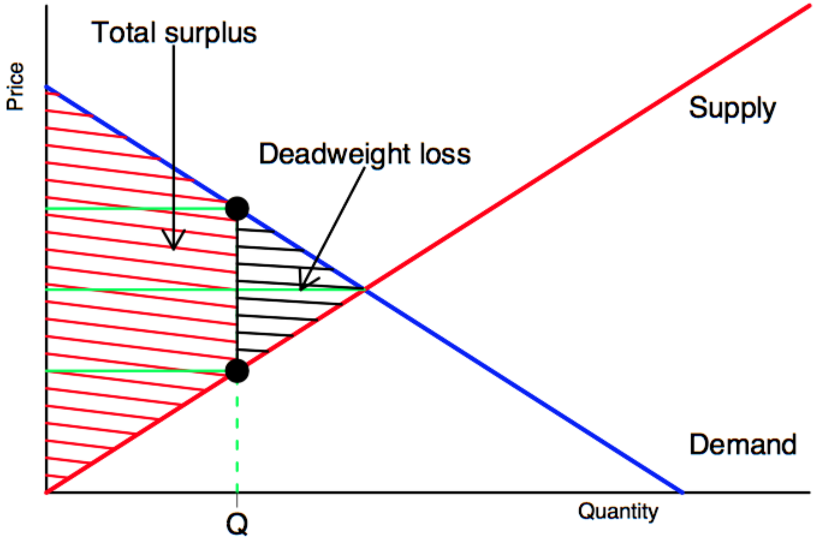
- Competitive equilibrium: the price p^* at which $D(p^*) = S(p^*)$
- Consumer surplus (CS) = willingness-to-pay (WTP) - price
- Producer surplus (PS) = price - marginal cost
- Social surplus = consumer surplus + producer surplus
- In the standard supply/demand diagram, the equilibrium is the point that maximizes the social surplus (the efficient point)
- If for some reason the quantity traded is different from the efficient quantity, there will be a deadweight loss (DWL)
- DWL = maximum social surplus - actual social surplus

Social surplus – graphical representation



Source: Gruber, *Public Finance and Public Policy*, Figure 2.16

Deadweight loss - graphical representation

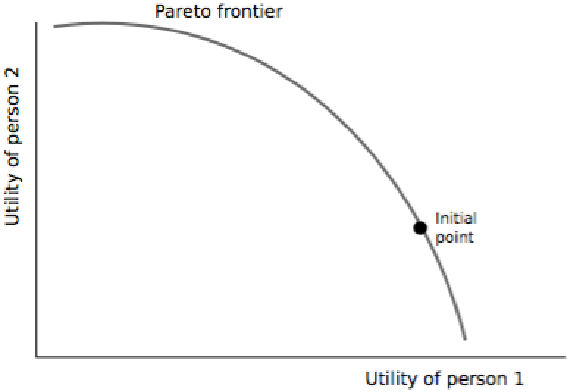


Pareto efficiency

- An allocation is Pareto efficient if the only way to make someone better off is to make another person worse
- If an allocation is not Pareto efficient, then there must exist a Pareto *improvement*
- At a Pareto efficient allocation the MRS is equal across all individuals (i.e. all the indifference curves must lie tangent to each other)
- This is not a statement about equality – an allocation can be efficient but highly unequal

Pareto efficiency does not imply fairness

- Pareto frontier gives the set of all efficient allocations
- Consider two individuals with the same utility function



Social welfare functions

If efficiency $\not\Rightarrow$ fairness, how do we “rank” individual utilities to pick a point on the Pareto frontier?

- A social planner picks an allocation, and thereby utility levels, to maximize a social welfare function $\mathcal{W}(U_1, \ldots, U_N)$
- Two common types of welfare functions:
  - Utilitarian: $U_1 + U_2 + \cdots + U_N$ (all utilities weighted equally)
  - Rawlsian: $\min\{U_1, U_2, \ldots, U_N\}$ (focus on least well-off person’s utility)
- In this course we will not take a stance on the form of the social welfare function beyond assuming it is concave
  - A concave $\mathcal{W}(\cdot)$ indicates the planner has a preference for some redistribution – she dislikes extreme inequality in utility levels

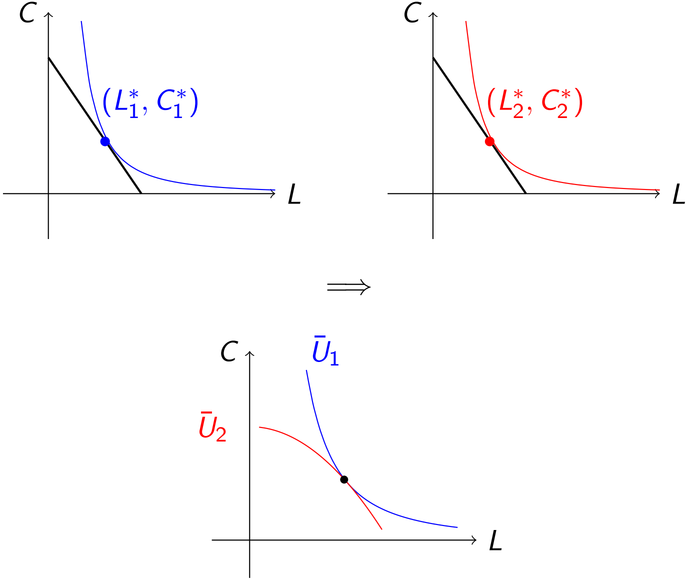
First Welfare Theorem

- A useful reference point that tells us the conditions under which "markets work":
  - No externalities
  - Perfect competition
  - Perfect information
  - Existence of markets for all goods
  - Rational individuals
- *If these conditions are met, any equilibrium allocation is Pareto efficient*
- Absent any redistribution motives, no need for government intervention if no market failures

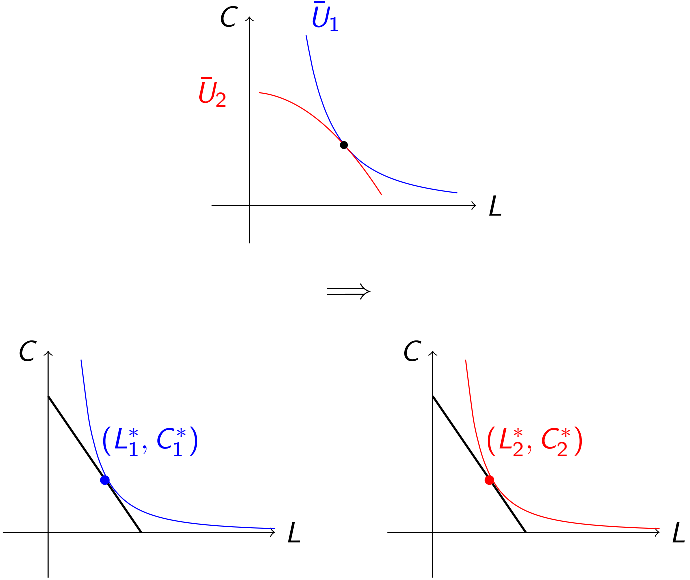
Second Welfare Theorem

- Another reference point that tells us there is no conflict between equity and efficiency
- *Any efficient allocation can be attained in equilibrium by redistributing initial endowments through lump-sum taxes/transfers*
- The taxes are based on individual characteristics and not behavior
- Requires some technical assumptions about preferences and goods production that we will not discuss here
- Main issue is that the required method of redistribution is unrealistic: how do you impose a lump-sum tax on innate talent?

First Welfare Theorem – illustration



Second Welfare Theorem – illustration



Summary

- Basic microeconomic tools needed to analyze how consumption and labor supply choice responds to taxes/transfers
  - Utility, budget constraints, optimization, demand/supply, equilibrium
- Key concepts: MRS, income and substitution effects, elasticity, Pareto efficiency, deadweight loss
- First and Second Welfare Theorems:
  - Failure of the First Welfare Theorem $\implies$ market failure – how can government intervention help?
  - Failure of the Second Welfare Theorem $\implies$ equity-efficiency trade-off because cannot redistribute initial endowments via lump-sum taxes